



# Propagation and amplification of tsunamis in the nearshore

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- ② Harbor Resonance
- ③ Trapped Waves around Islands
- ④ Trapped Waves over Ridges
- ⑤ Further Research

# 1 Introduction

**Tsunamis:** Japanese term for the large waves in harbors ( tsu = harbor, nami = wave ).

津波(つ なみ)

Tsunamis were previously called tidal waves, but are unrelated to tides.

Tsunamis are also seismic sea waves.



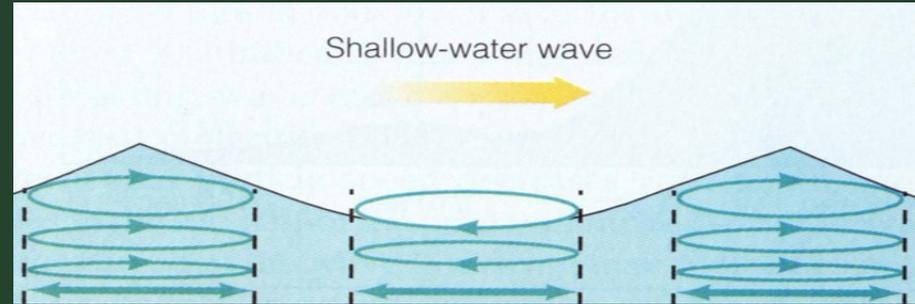
# 1 Introduction

- Tsunamis  
(landslides, volcanic activities, earthquakes)
- Meteorological Tsunamis  
(atmospheric gravity waves, pressure jumps, frontal passages, squalls, meteorological waves)

# 1 Introduction

## Tsunamis

- Shallow water waves



- Fast speed in deep water

$$c = c_g = \sqrt{gh}$$



- Small amplitude in ocean and large amplitude nearshore



# 1 Introduction

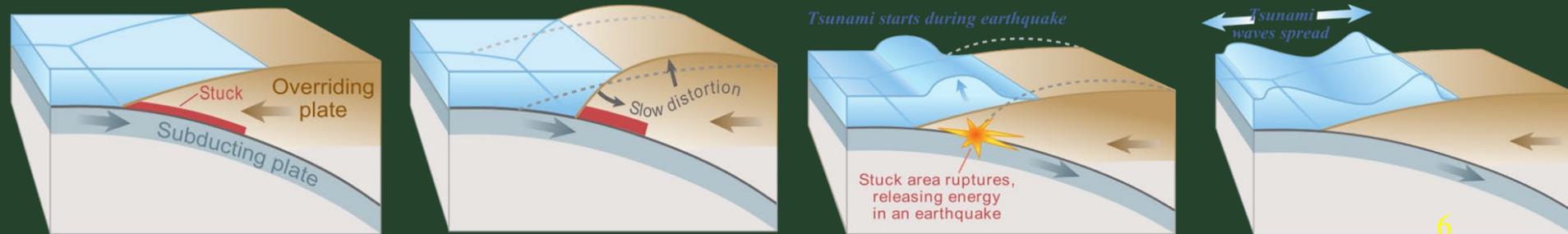
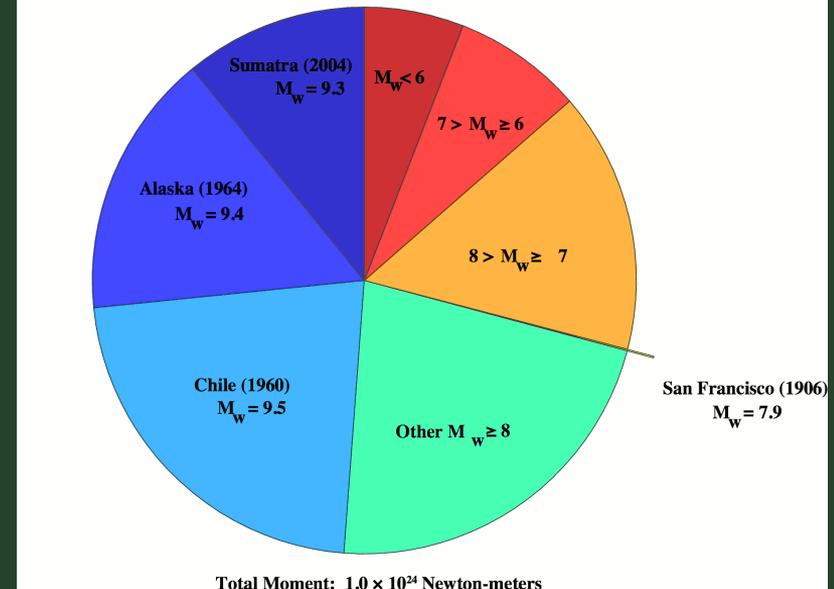
## Tsunamis

Fault displacement with a large earthquake

(they are sometimes called “seismic sea waves” )

- Damaging tsunami in the near field (magnitude  $> 7$ )
- Tsunami in the far field (magnitude  $> 8.0$ )

Global Seismic Moment Release January 1906 - December 2005



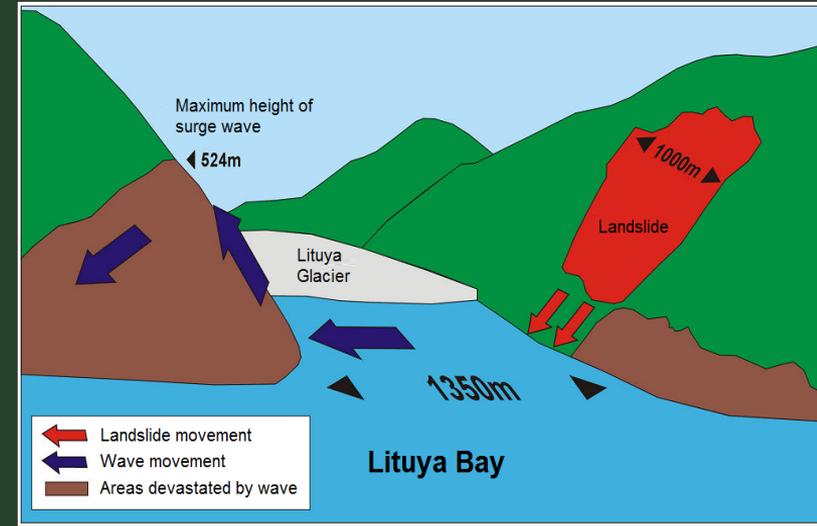
# 1 Introduction

## Tsunamis

Landslide – into or below the water surface

The 1958 Lituya Bay, Alaska

- 30 million cubic meters of rock slides
- Slide area was approx. 800 m by 900 m.
- Generated a 524 m splash-up immediately across the bay.
- Note quick attenuation of tsunami height.



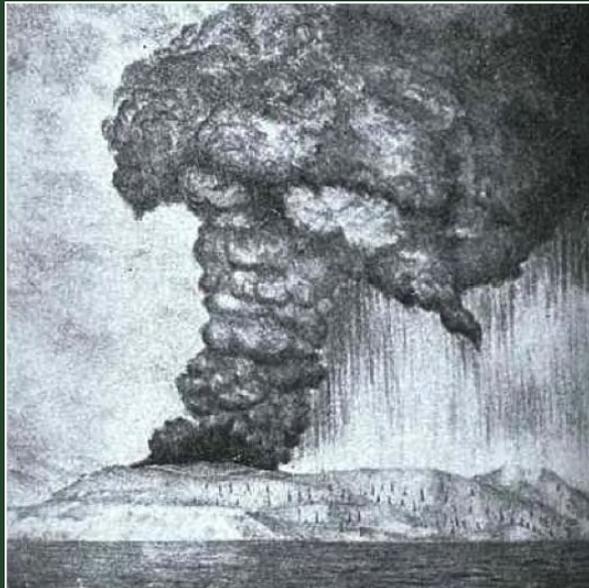
# 1 Introduction

## Tsunamis

### Volcanic Eruption

The 1883 eruption of Krakatau, Indonesia:

- The largest wave runup height, 40 meters and killed over 36,500 people,
- The island dimension is approx. 5 km in diameter.



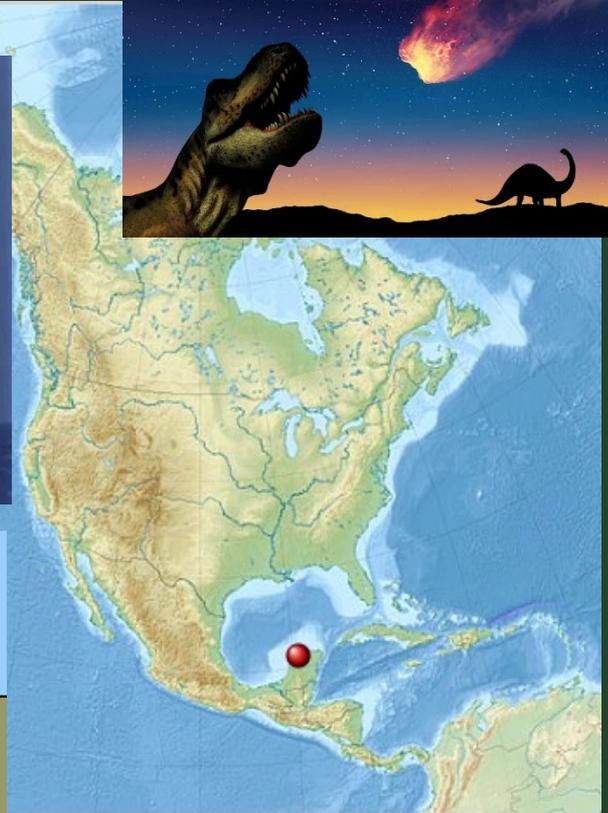
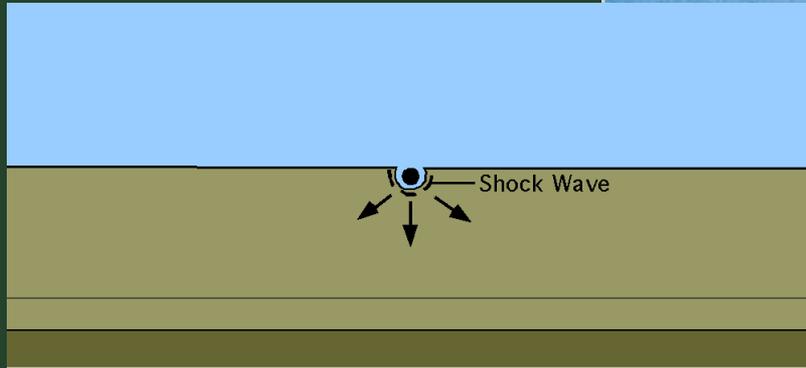
# 1 Introduction

## Tsunamis

Nuclear explosion tsunami



Planet tsunami

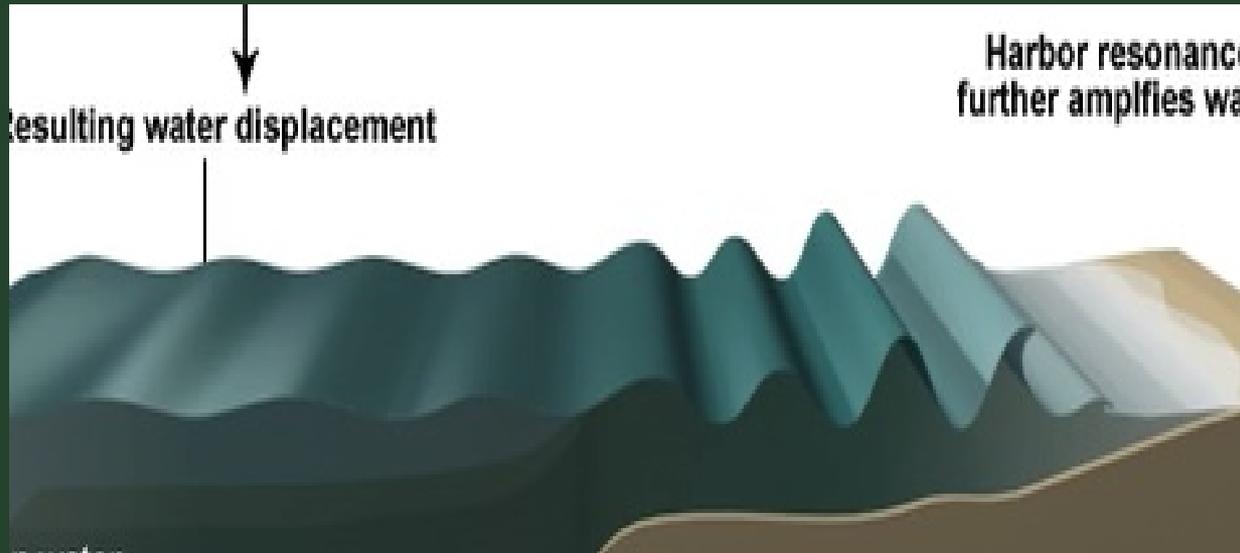


Chicxulub crater, Mexico

# 1 Introduction

## Meteorological Tsunamis

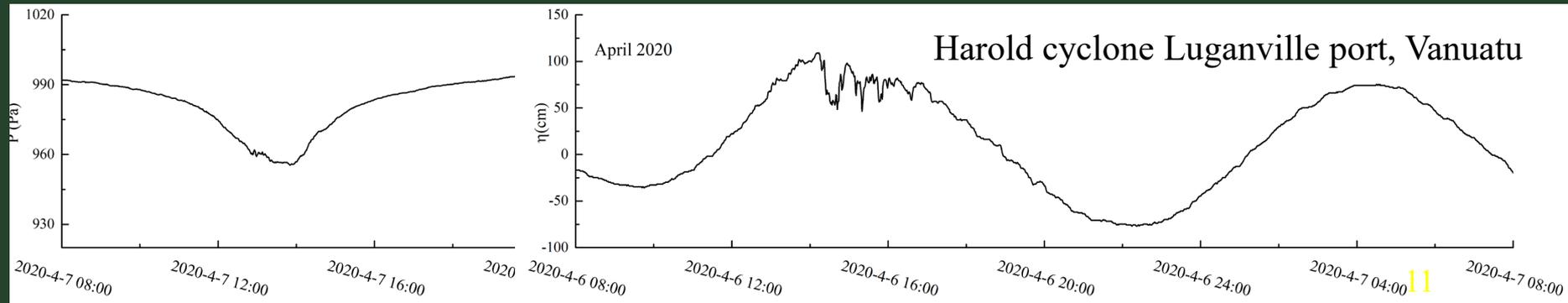
Water-level oscillations which are similar to waves generated by seismic activity ('tsunami waves'), except they have a meteorological origin and are not generated through seismic activity, volcanic explosions or submarine landslides



# 1 Introduction

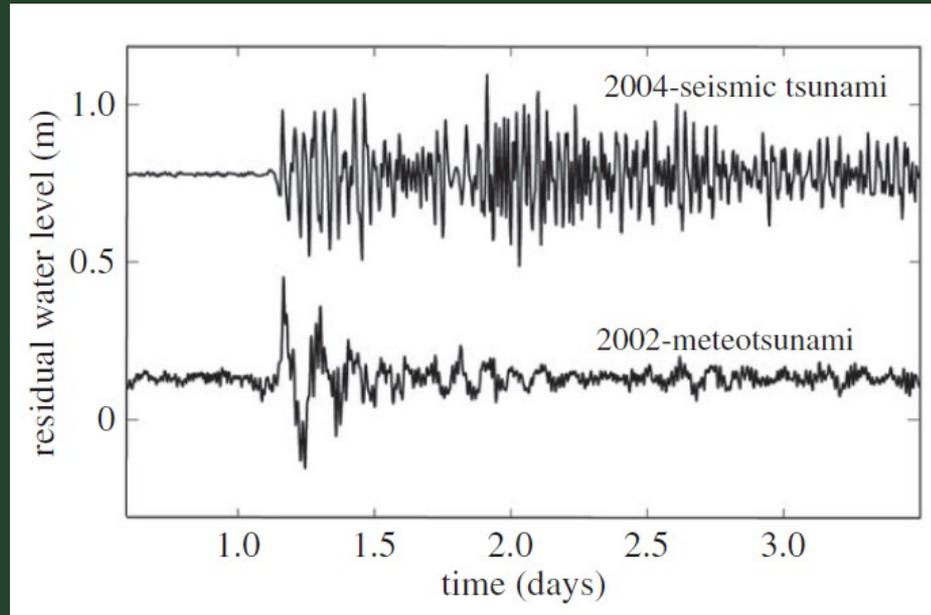
## Meteorological Tsunamis

- Meteorological tsunamis are primarily caused by the propagation of an abrupt atmospheric pressure change and associated wind gusts.
- There must be resonance between the atmospheric disturbance and the wave speed in the deep water.
- These waves are usually small in open sea, and amplified through nearshore resonance, such as harbor resonance.



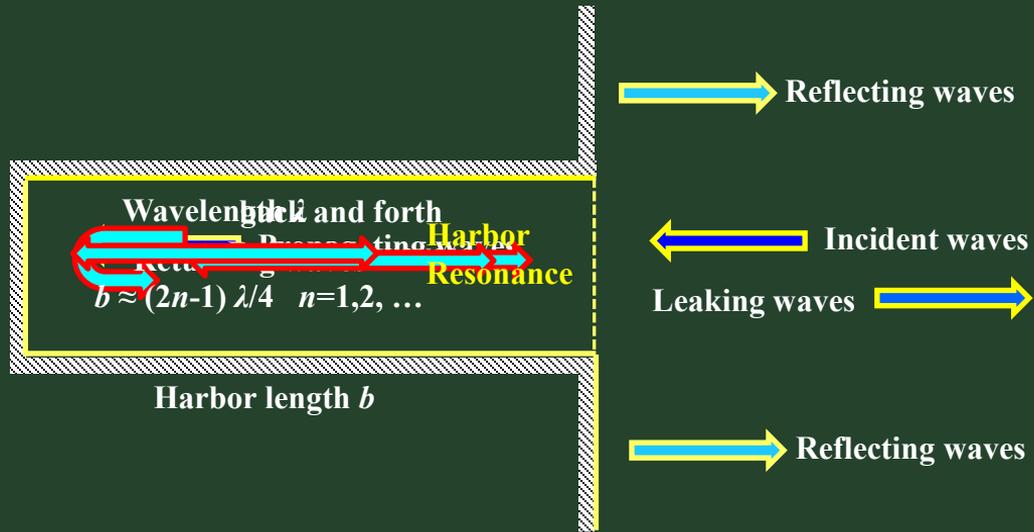
# 1 Introduction

## Meteorological Tsunamis



Relatively small initial sea-level perturbations, of the order of a few centimeters, can increase significantly through multiresonant phenomena to create destructive events through the superposition of different factors.

## 2 Harbor Resonance



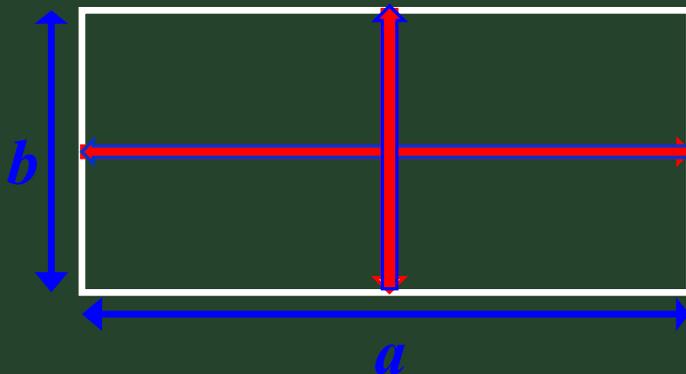
Seiches

Harbor resonance is such kind of water disasters that large oscillations are induced by incident waves with frequency close to one of the natural frequencies of the harbor.

When those waves propagating back and forth within the harbor satisfy a certain relationship, the resonance is resulted in and there are large oscillations in the harbor.

# 2 Harbor Resonance

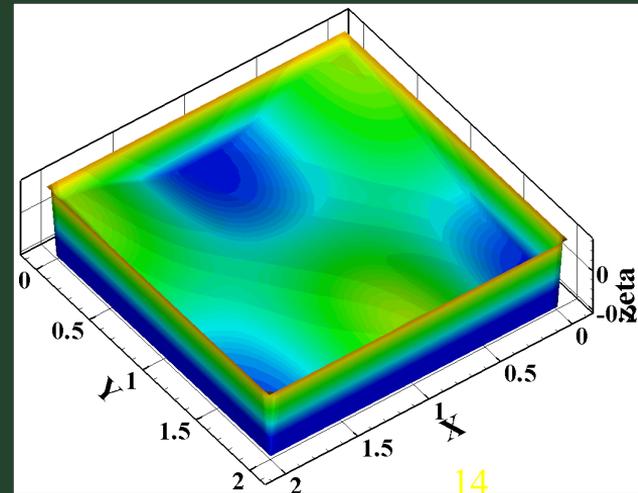
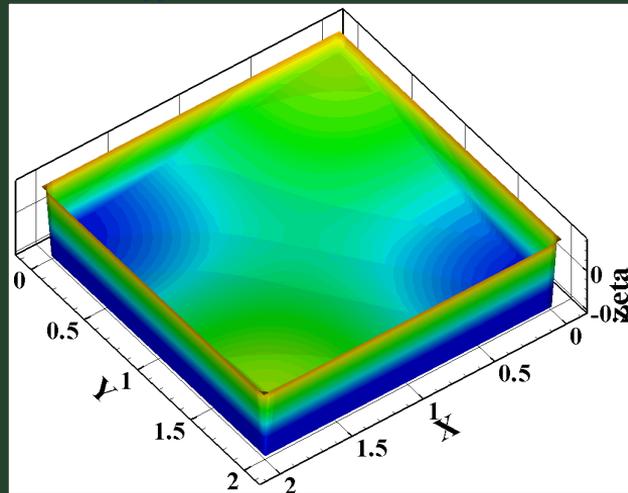
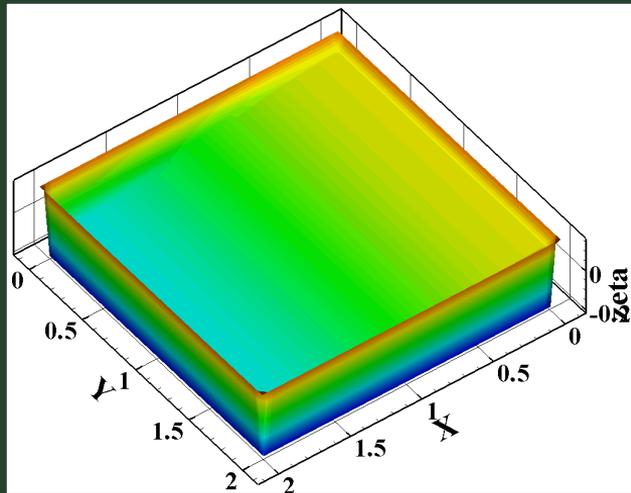
wave ray  
forms a  
close loop



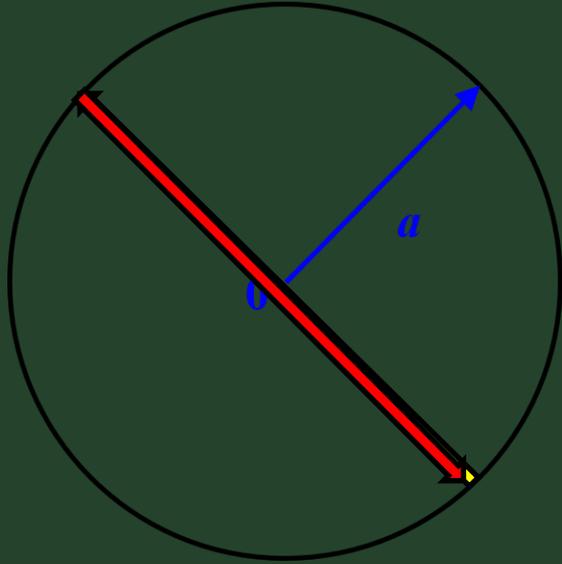
$$\eta = A_{nm} \cos \frac{n\pi x}{a} \cos \frac{m\pi y}{b} \cos \omega_{nm} t$$

$$k_{nm} = 2\pi/\lambda = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$$

$$\omega_{nm} = 2\pi/T_m \quad \omega_{nm} = k_{nm} \sqrt{gh}$$



# 2 Harbor Resonance

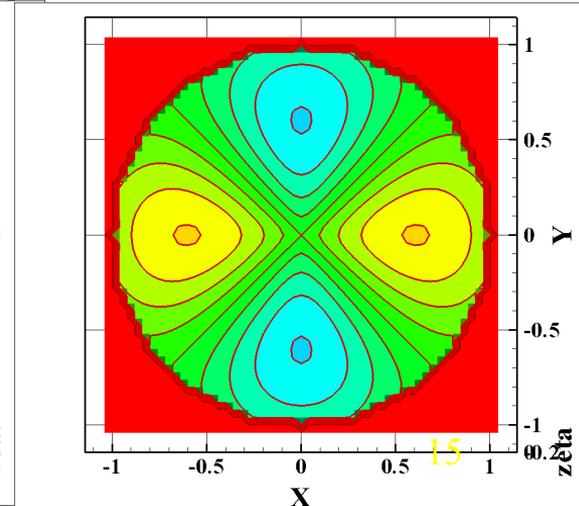
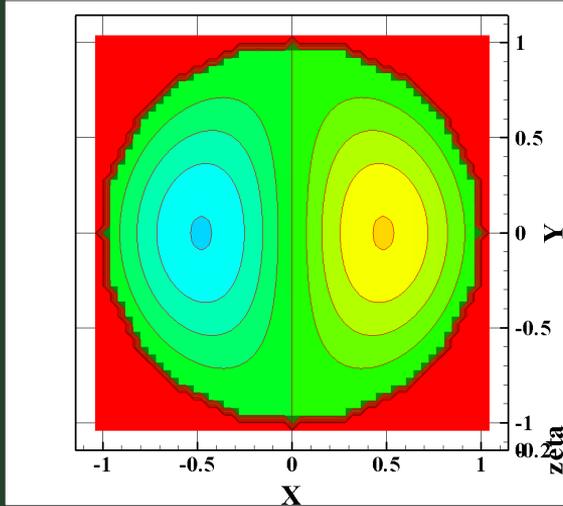
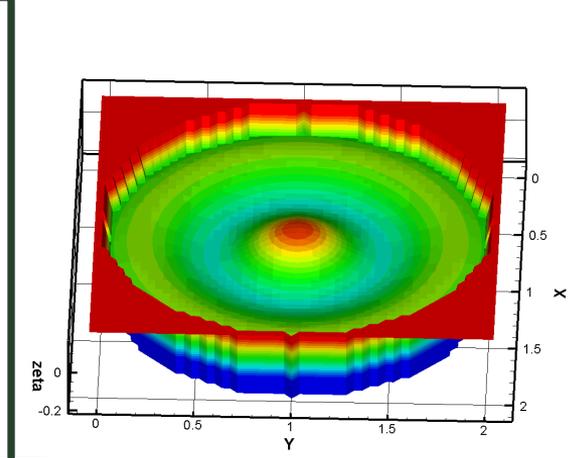
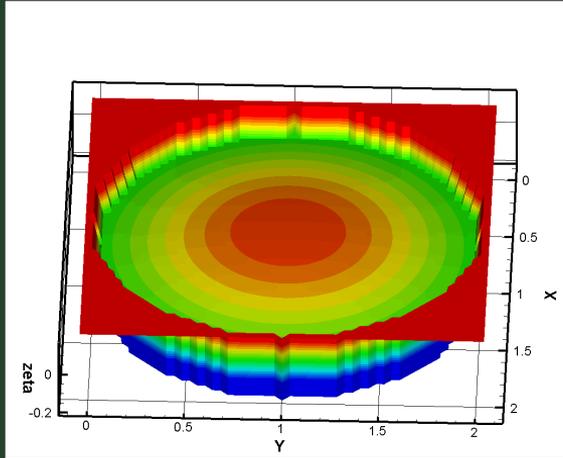


$$\eta_{nm} = (A_{nm} \cos m\theta + B_{nm} \sin n\theta)$$

$$\cdot J_{nm}(k_{nm}r) \cos \omega_{nm}t$$

$$\omega_{nm} = 2\pi/T_m \quad \omega_{nm} = k_{nm} \sqrt{gh}$$

**wave ray forms a close loop**



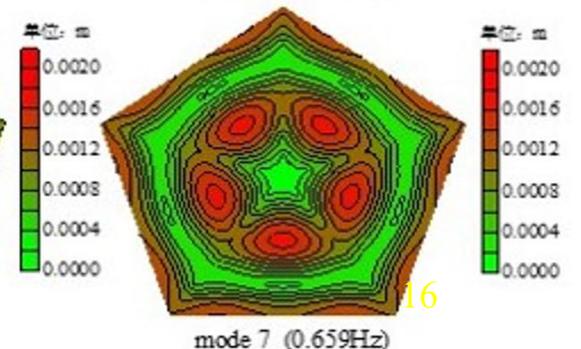
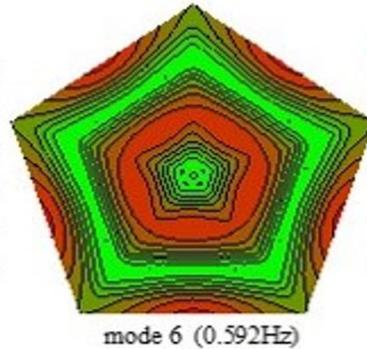
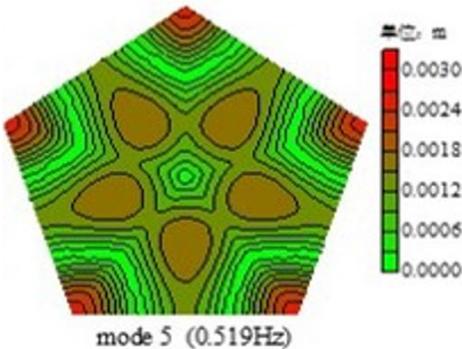
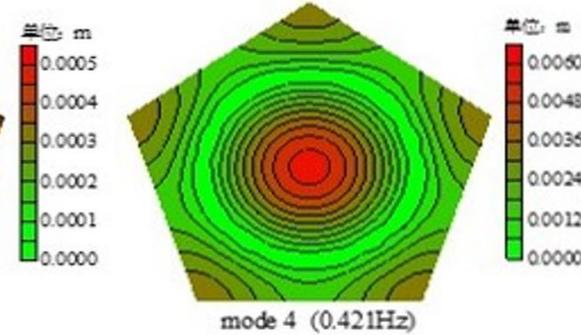
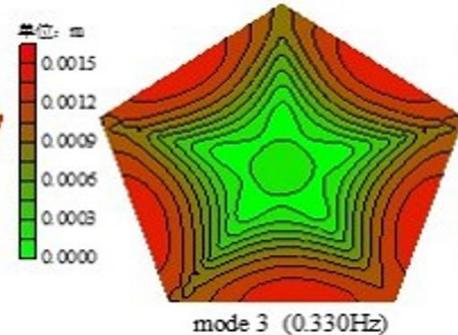
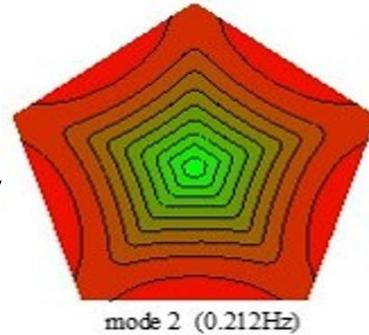
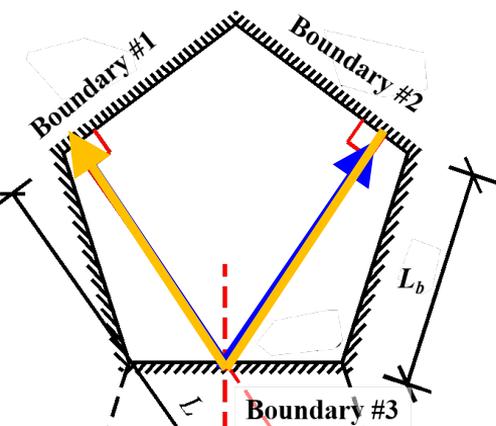
# 2 Harbor Resonance

## regular pentagon basin

$$L = 2 \cos(\pi/5) [\cos(\pi/10) + \cos(3\pi/10)] L_b$$

$$\omega_n = \frac{n\pi\sqrt{gh}}{2L_b \cos(\pi/5) [\cos(\pi/10) + \cos(3\pi/10)]}$$

$$k_n = \frac{n\pi}{L}$$
$$n = 1, 2, 3, \dots$$



Based on the principle that standing waves are formed when the wave propagation route is a loop due to the multiple reflections at the boundaries, we propose analytical solutions of the eigen values for the regular-pentagon basin.

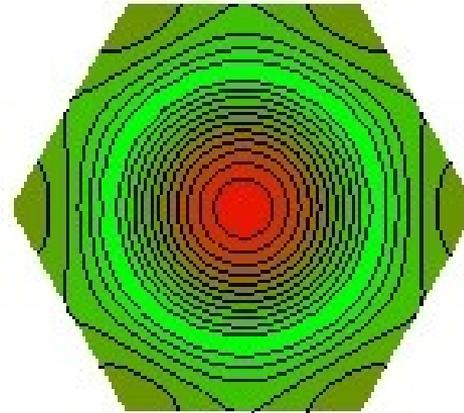
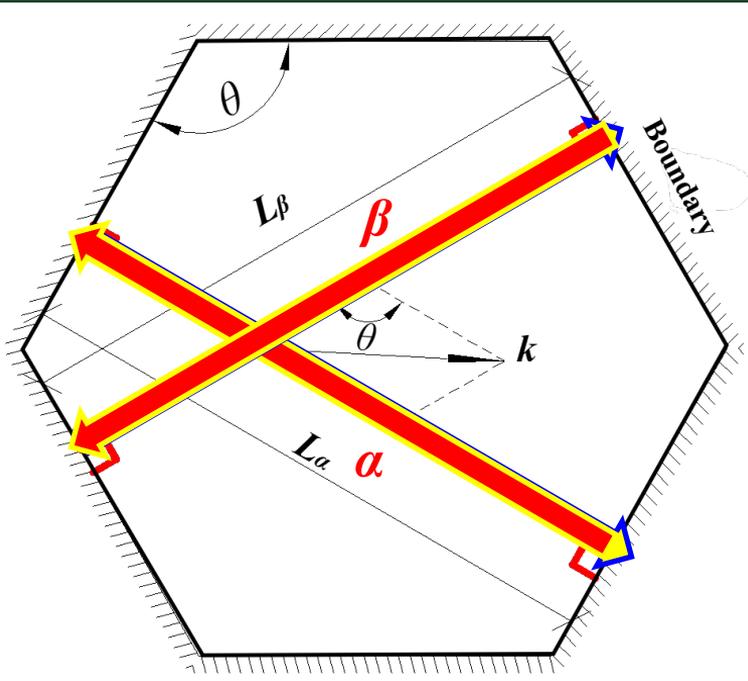
# regular hexagon basin

For oscillations in a regular hexagon, the wave is multiple reflected between the two opposite sides.

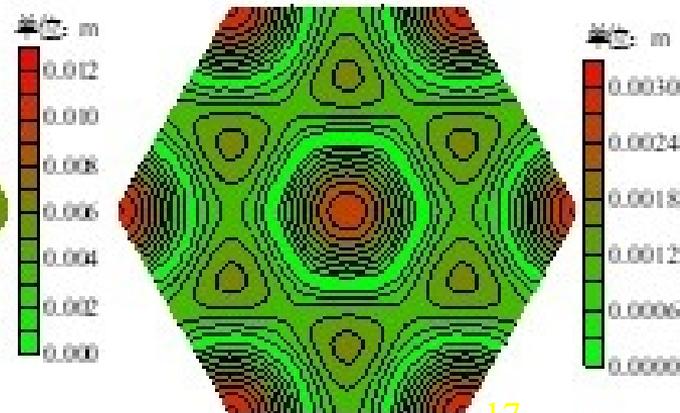
$$k_m = \frac{m\pi}{L_\alpha}, k_n = \frac{n\pi}{L_\beta} \quad m, n = 1, 2, 3, \dots$$

$$\vec{k}_{mn} = k_m \vec{e}_\alpha + k_n \vec{e}_\beta \quad |k_{mn}| = \sqrt{k_m^2 + k_n^2 - 2k_m k_n \cdot \cos(2\pi/3)}$$

$$\omega_{mn} = \sqrt{gh \left[ k_m^2 + k_n^2 - 2k_m k_n \cdot \cos(2\pi/3) \right]}$$

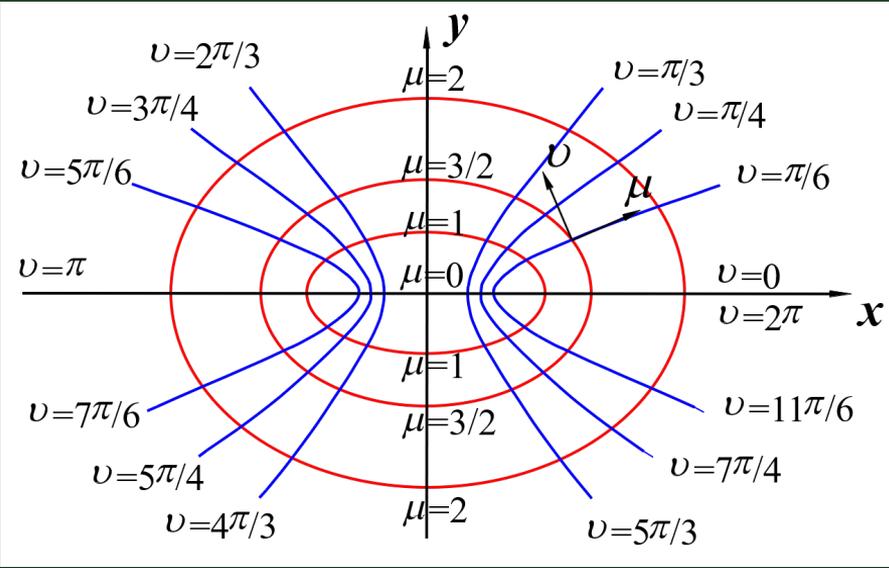


mode (2,0) (0.305Hz)



mode (4,0) (0.568Hz)

# elliptical basin



## Shallow water equations in the elliptic coordinate

$$x = \alpha \cosh(\mu) \cos(\nu) \quad y = \alpha \sinh(\mu) \sin(\nu)$$

$$\nabla^2 \zeta + k^2 \zeta = 0 \quad k = \omega / \sqrt{gh}$$

$$\zeta_{\mu\mu} + \zeta_{\nu\nu} + \frac{k^2 \alpha^2}{2} (\cosh 2\mu - \cos 2\nu) \zeta = 0$$

## Separation of variables

$$\zeta(\mu, \nu) = M(\mu)N(\nu)$$

the ordinary Mathieu equation

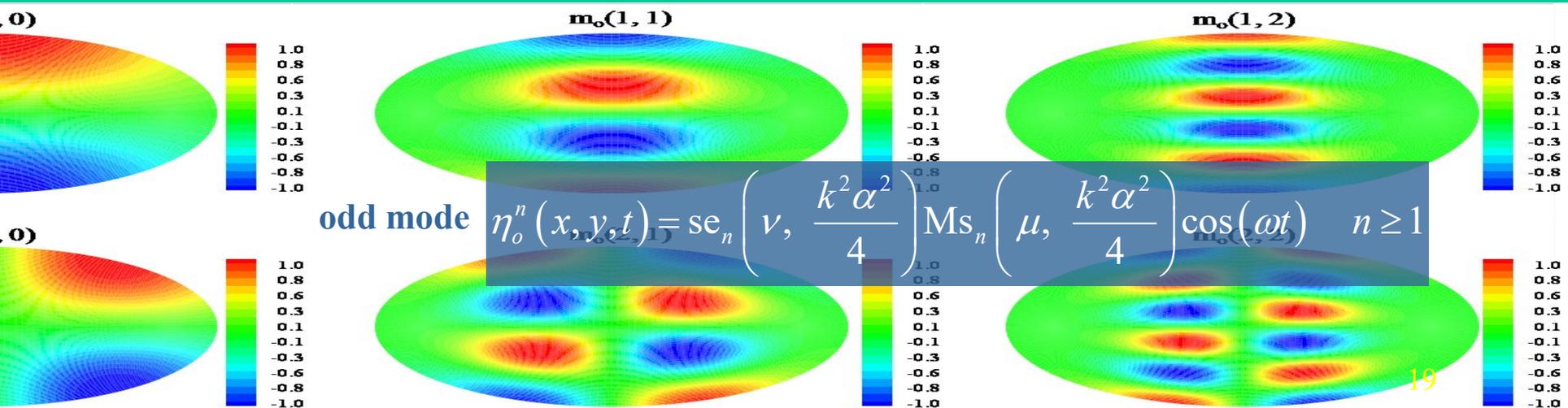
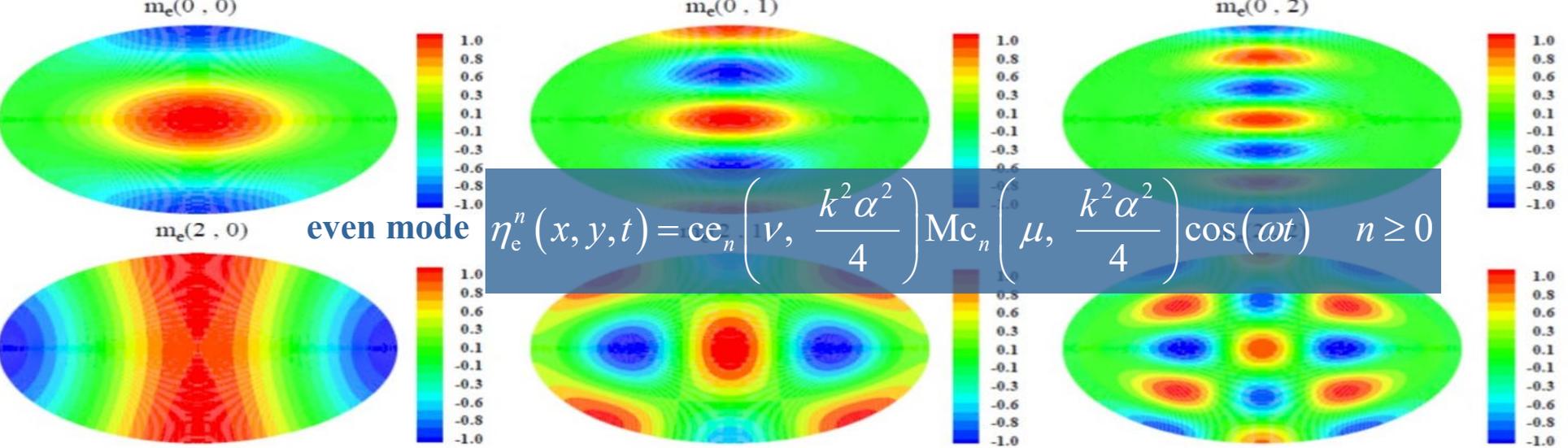
$$N'' + \left( \lambda - \frac{k^2 \alpha^2}{2} \cdot \cos 2\nu \right) N = 0$$

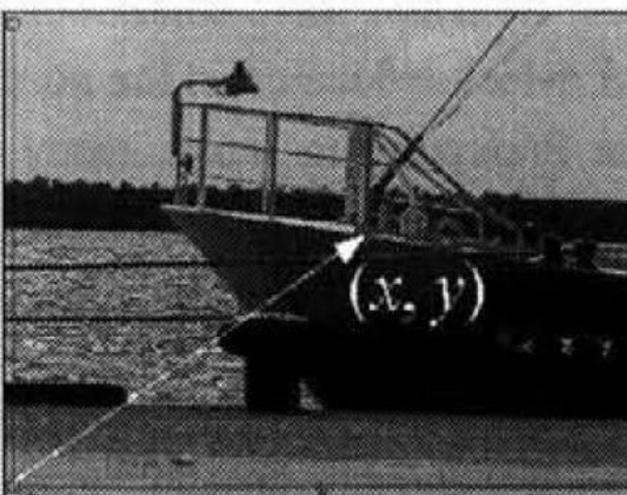
the modified Mathieu equation

$$M'' - \left( \lambda - \frac{k^2 \alpha^2}{2} \cdot \cosh 2\mu \right) M = 0$$

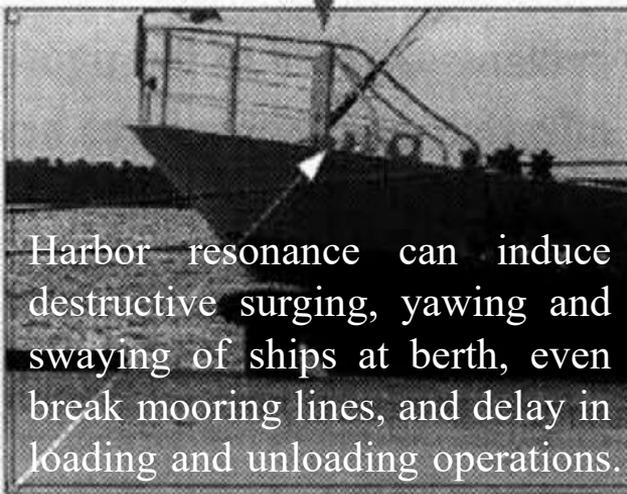
Eigenvalues are obtained with the non-flux boundary condition.

$$\frac{\partial \zeta}{\partial \mu} = 0 \quad \text{where } \mu_b = \operatorname{arccosh} \frac{a}{\sqrt{a^2 - b^2}}$$

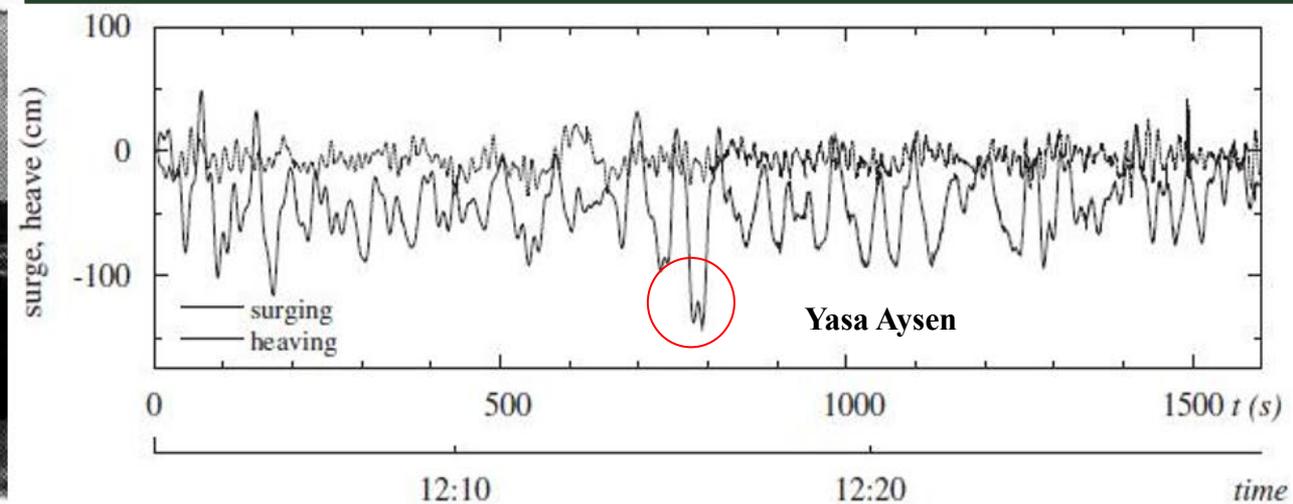




6 s later

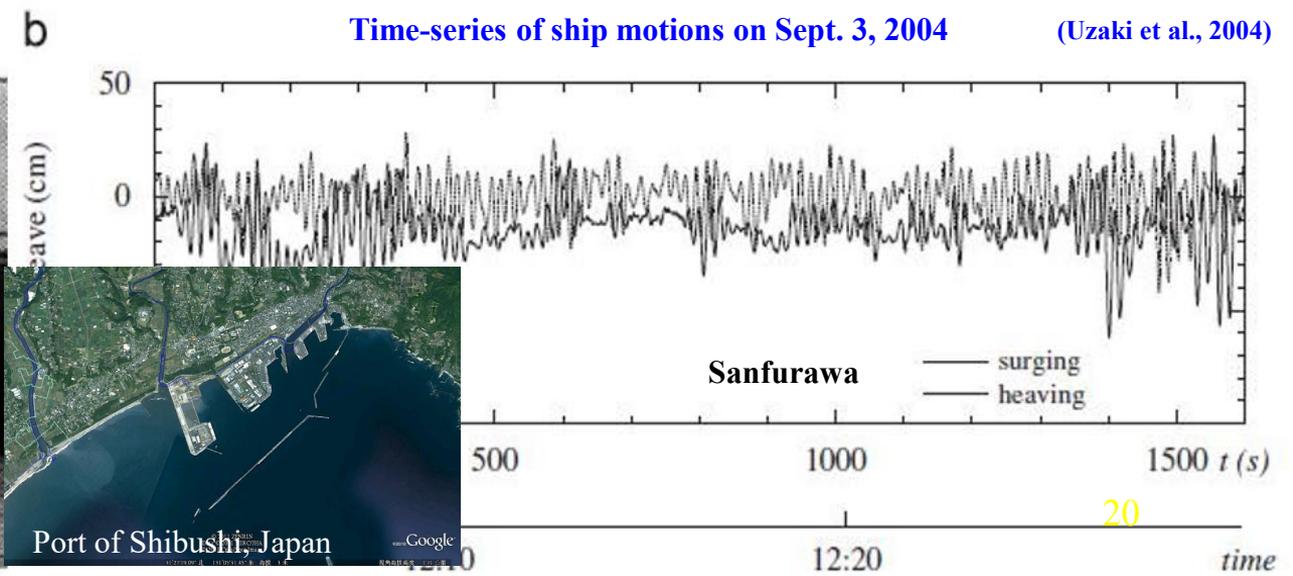


Harbor resonance can induce destructive surging, yawing and swaying of ships at berth, even break mooring lines, and delay in loading and unloading operations.



**Time-series of ship motions on Sept. 3, 2004**

(Uzaki et al., 2004)



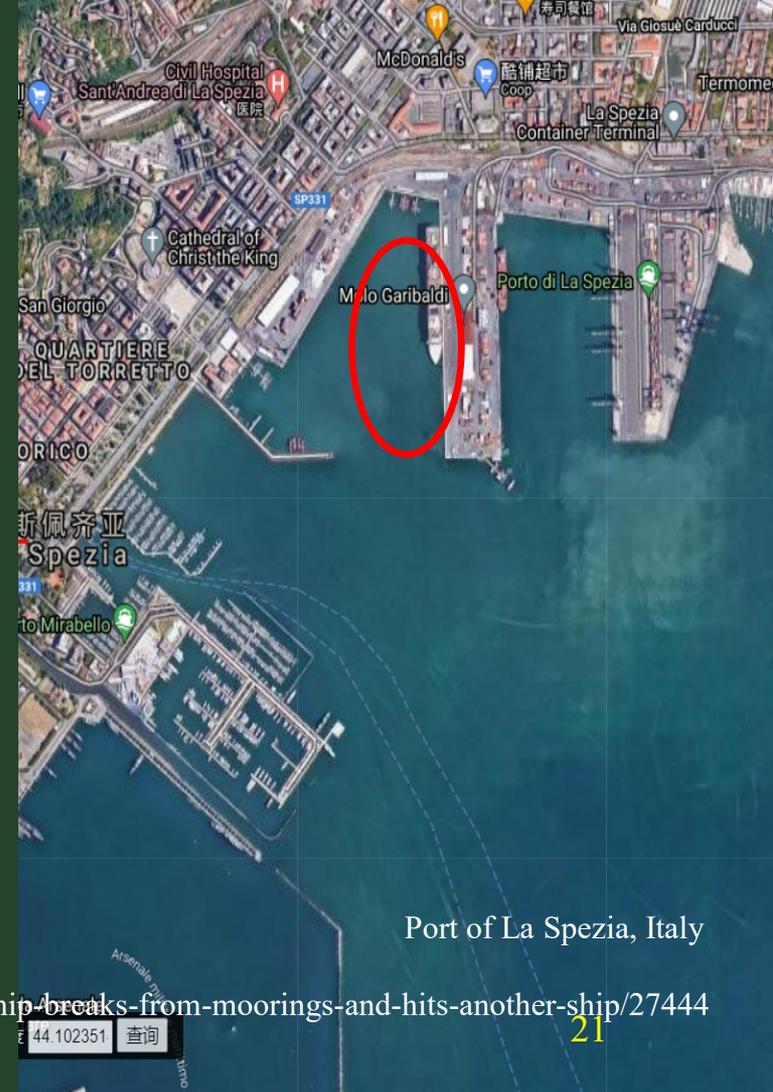
Port of Shibushi, Japan

Google



Celebrity Cruise Ship broke from moorings, Oct. 29, 2018

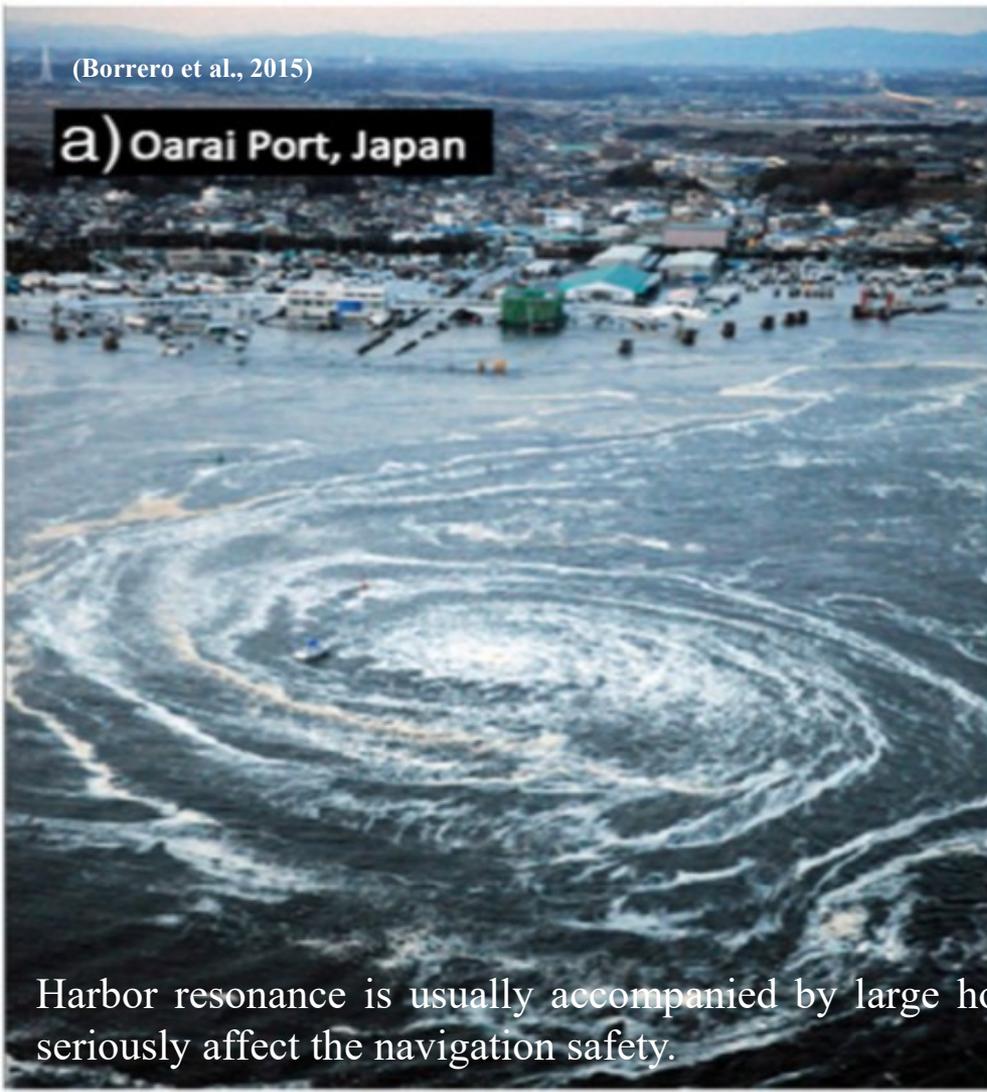
<https://www.cruisehive.com/cruise-ship-breaks-from-moorings-and-hits-another-ship/27444>



Port of La Spezia, Italy

(Borrero et al., 2015)

**a) Oarai Port, Japan**



(Lynett et al., 2012)

**b) Crescent City Harbor, USA**



**c) Pillar Point Harbor, USA**

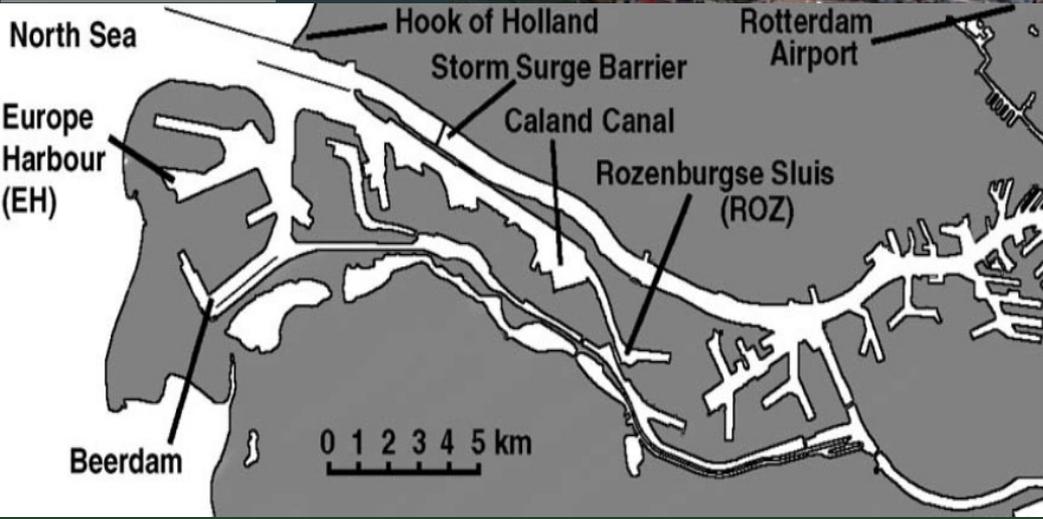


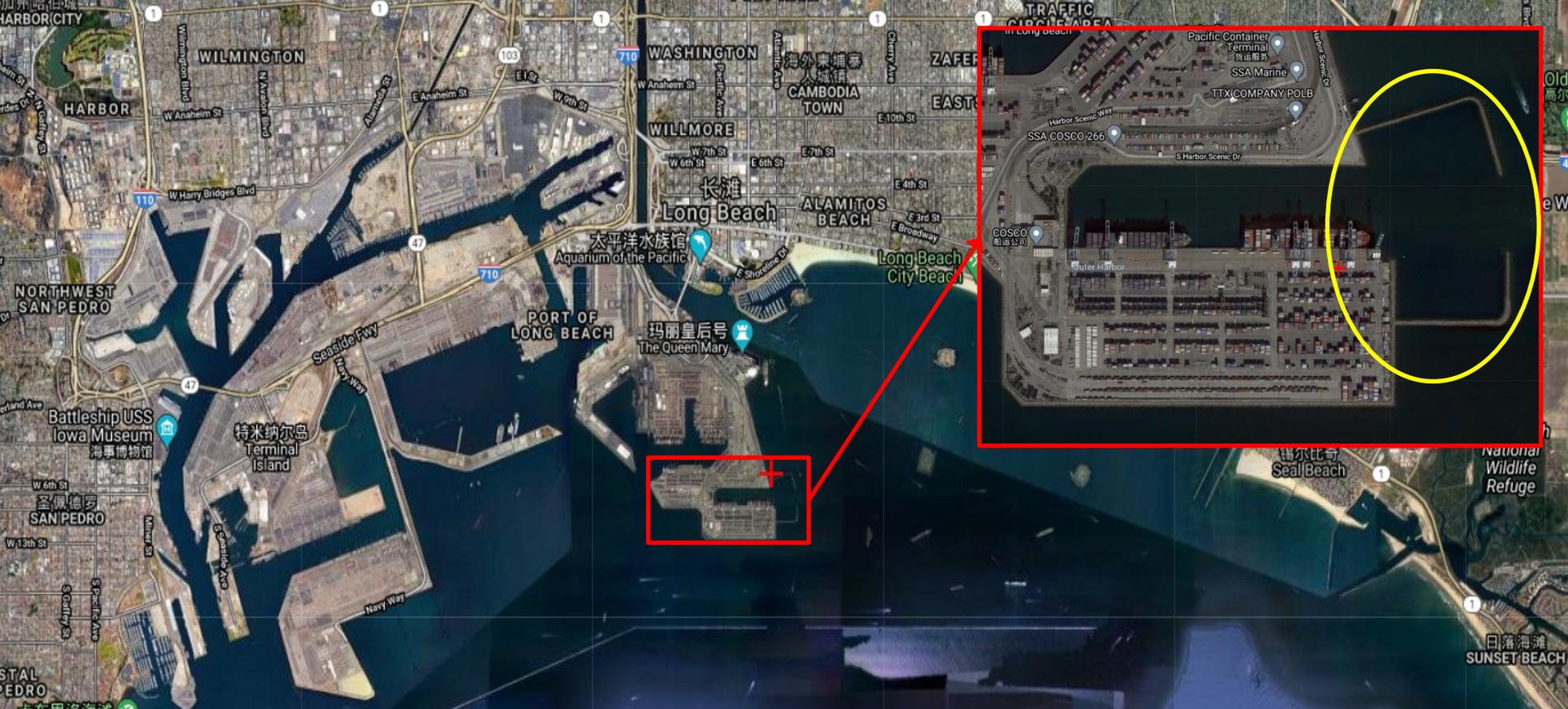
Harbor resonance is usually accompanied by large horizontal water motions at the entrance, which can seriously affect the navigation safety.



Port of Yulin, China







The eigen periods and associated modal structures are a fundamental property of a harbor and determined by the geometry and topography of the basin. Changing configuration will lead to variation of the eigenvalue of the harbor. The vessel surge motions are attenuated by extending the breakwaters near the entrance of the Pier J in Port of Long Beach .



West Baltic

Western Ireland  
Rotterdam Harbor

Marseilles  
Gulf of Trieste  
Croatian coast  
Toulon  
Ciudadella & Platja Gran  
Weat Sicily  
Alger  
Malta  
Madeira Islands  
Batumi

Dakar

Table Bay

Salalah

Colombo Harbor

Trincomalee  
Hambanthota Harbor

Bunbury Harbor  
Esperance Bay

Longkou Harbor

Hua-Lien Harbor

Puerto Princessa

Pohang Harbor  
Malokurilsk Bay  
Tomakomai Harbor

Oarai Harbor

Port of Shibushi

Nagasaki Bay

Skagwany

Albemi

Crescent

Port of Long Beach

Barbers Point  
Hilo

Manzanillo

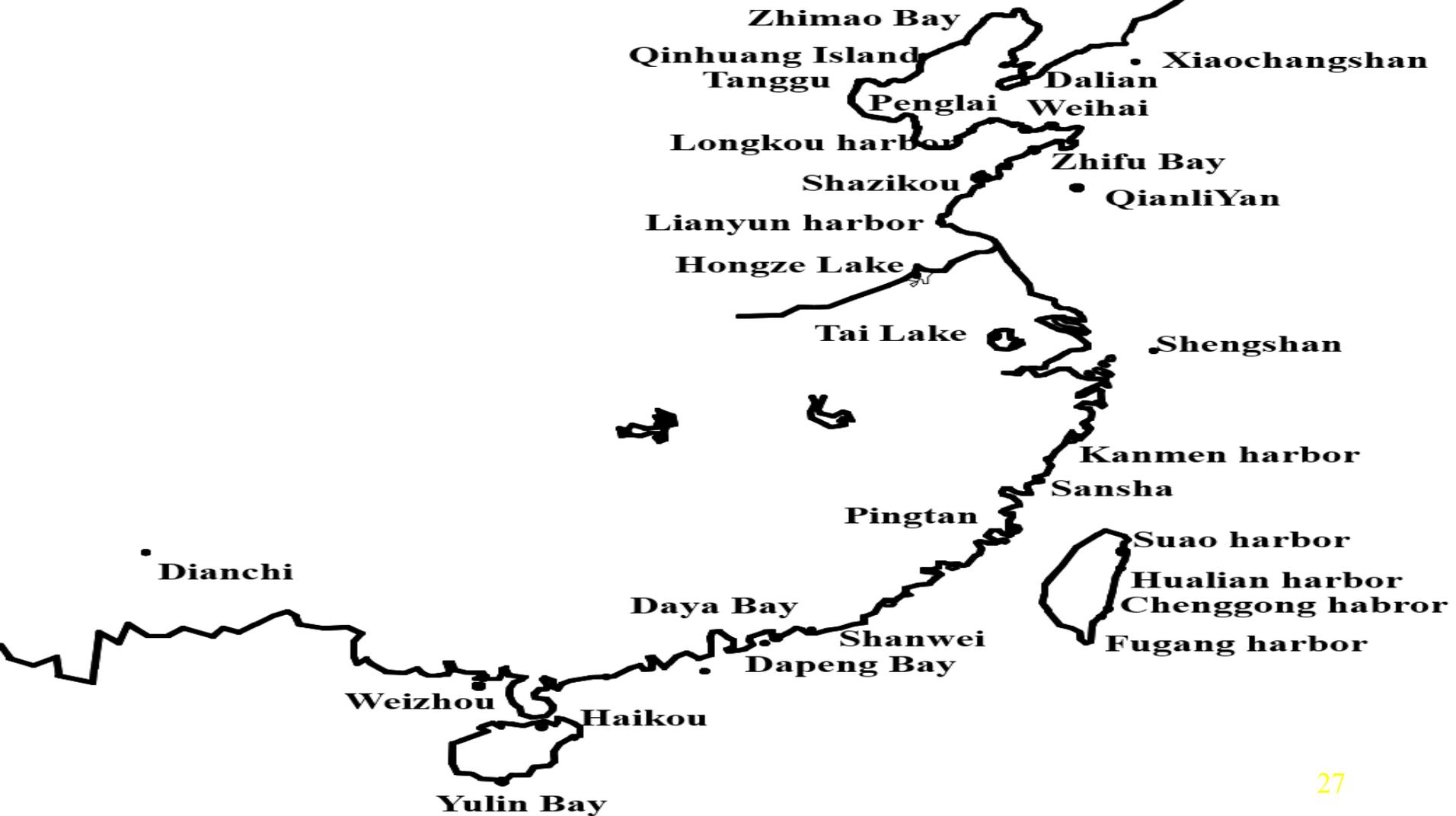
Newfoundland

Azores Islands

Tauranga Harbor

Wellington Harbor

Talcahuano





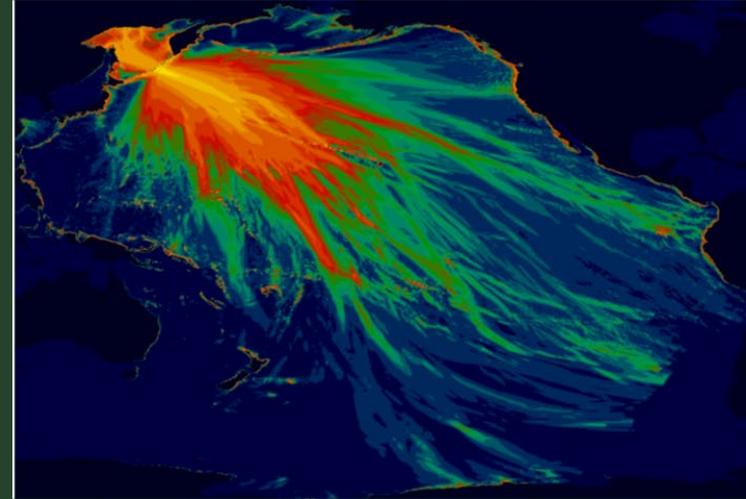
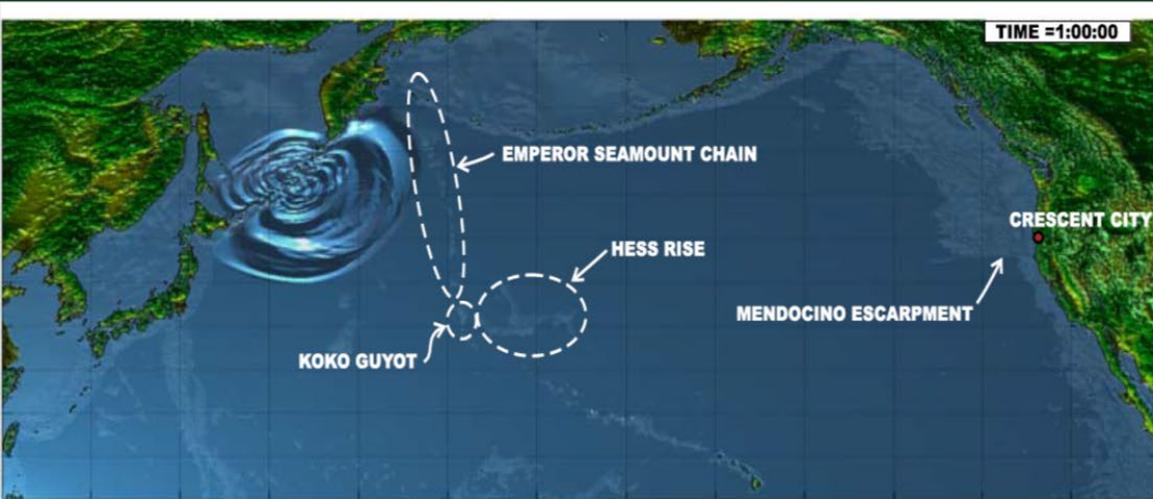
## 2 Harbor Resonance



The resonance phenomenon has been mitigated after a series of offshore breakwaters constructed.

Physical experiments and numerical simulations confirm that evident edge waves are formed during the typhoon, and they are responsible for the resonance of Hua-Lien Harbor.

## 2 Harbor Resonance



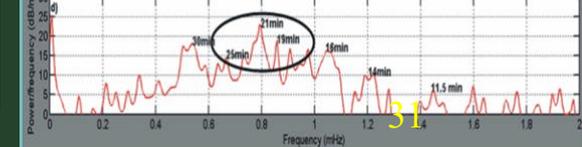
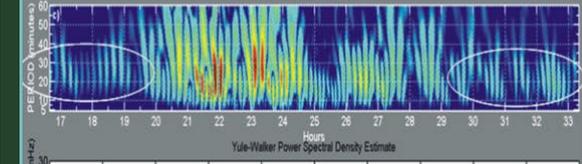
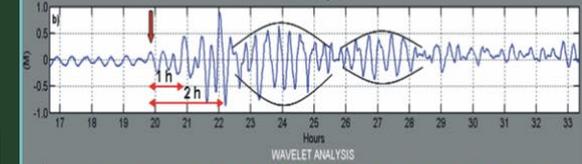
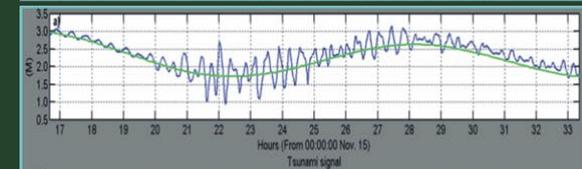
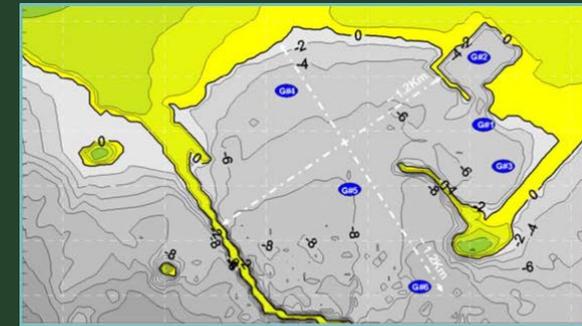
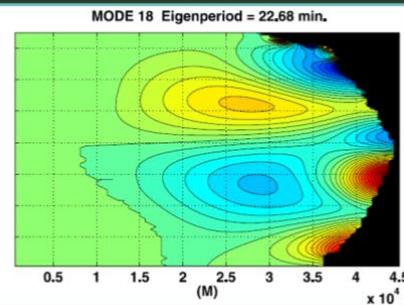
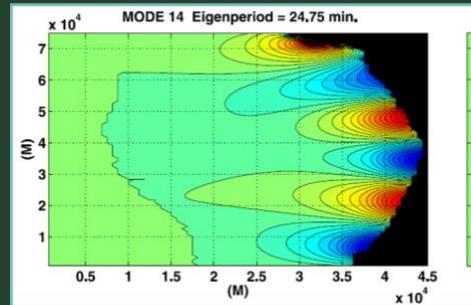
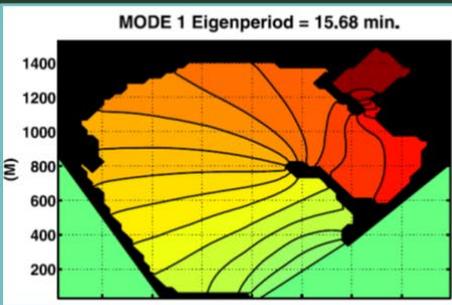
On 15 November 2006 at 11:14:16 (UTC) an earthquake with moment magnitude 8.3 generated a tsunami near the Kuril Islands.

Tsunamis propagated over the entire Pacific Ocean.

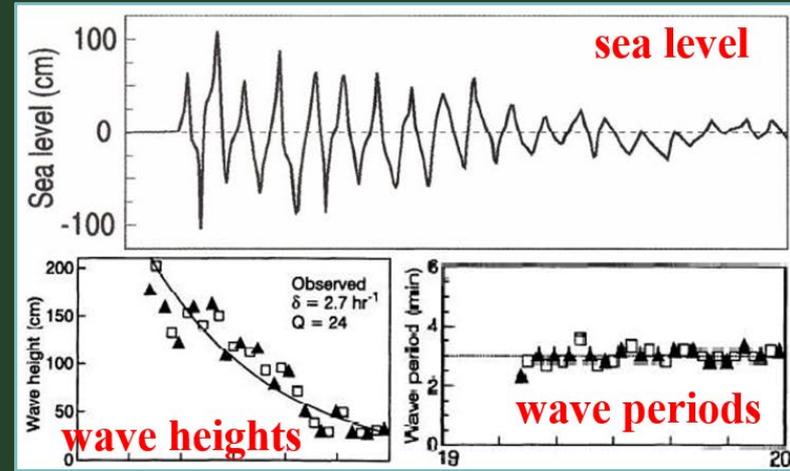
The small initial tsunami of about 20 cm was barely noticed at Crescent City. The highest wave of about 88 cm amplitude was recorded 2–3 hours later.

# 2 Harbor Resonance

- Crescent City tide gauge record of 15 November 2006 revealed that there are oscillations with periods from 10 to 30 min in the harbor.
- Harbor resonance with period of O(10min.) is excited.
- Shelf standing waves with periods of ranged from 20 to 30 min. Are present, which enhanced the oscillations within the harbor.

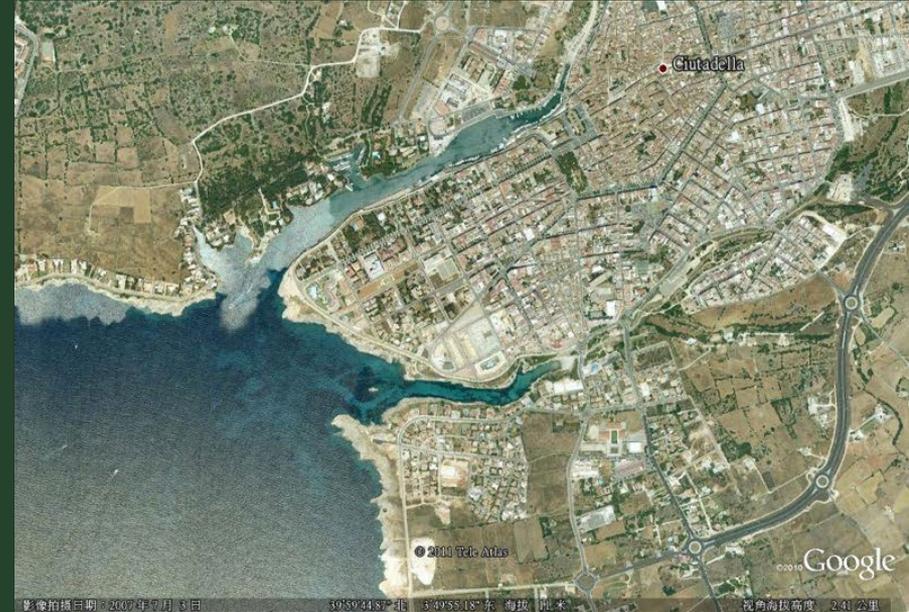
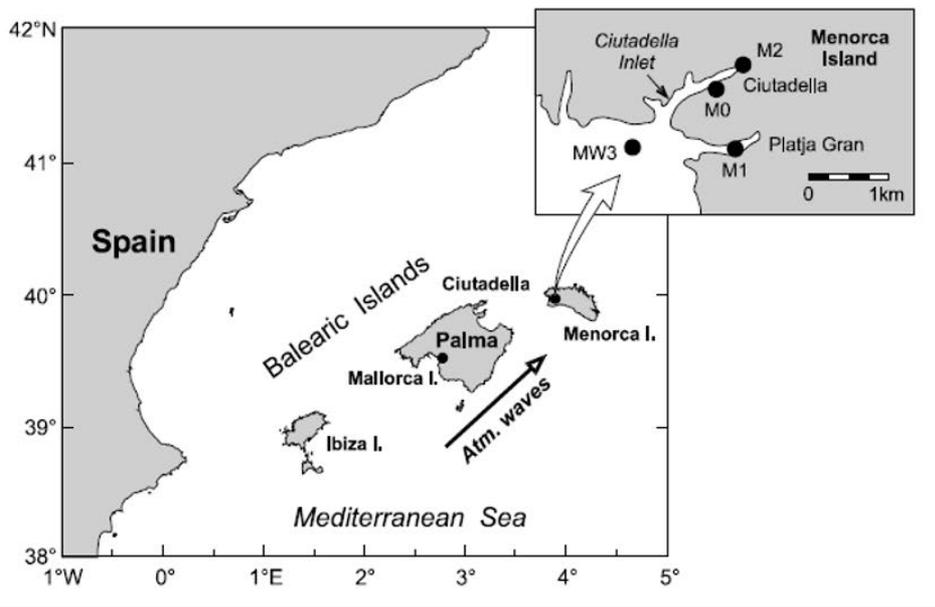


# 2 Harbor Resonance



- The tsunami of November 3, 1994 in Skagway, Alaska, was generated by an underwater landslide formed during the collapse of a cruise ship wharf undergoing construction at the head of Taiya Inlet.
- Persistent wave motions with an amplitude of 1 m and a period of 3 min. are recorded by a tide gauge in the harbor, which is supposed that the first mode resonance is excited.

# 2 Harbor Resonance

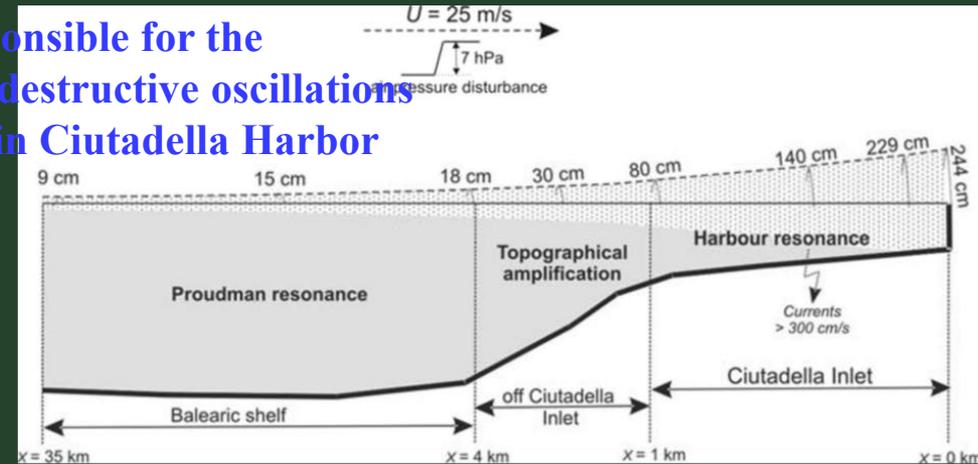


- Ciutadella Inlet is about 1 km long, 100m wide, and 5m deep; the harbor is located at the head of the inlet.
- These significant short-period sea level oscillations induced by meteorological tsunamis regularly occur in Ciutadella Harbor, Menorca Island.

## 2 Harbor Resonance



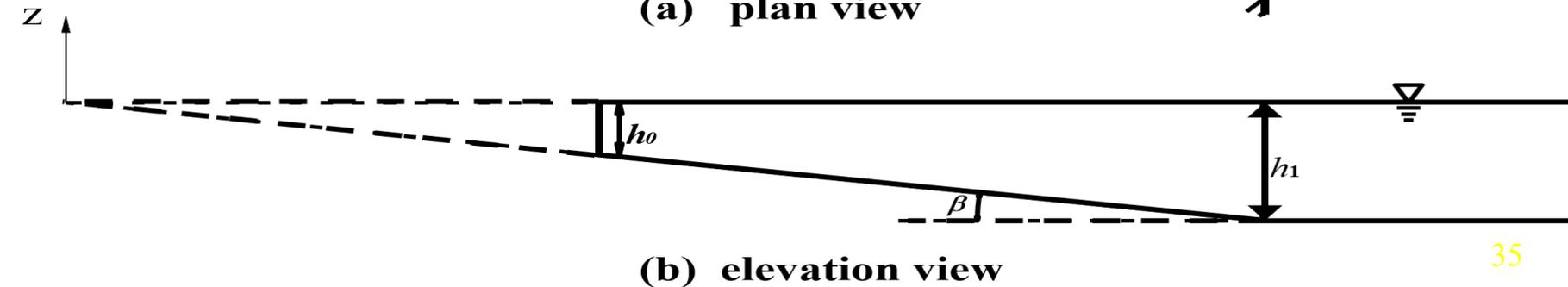
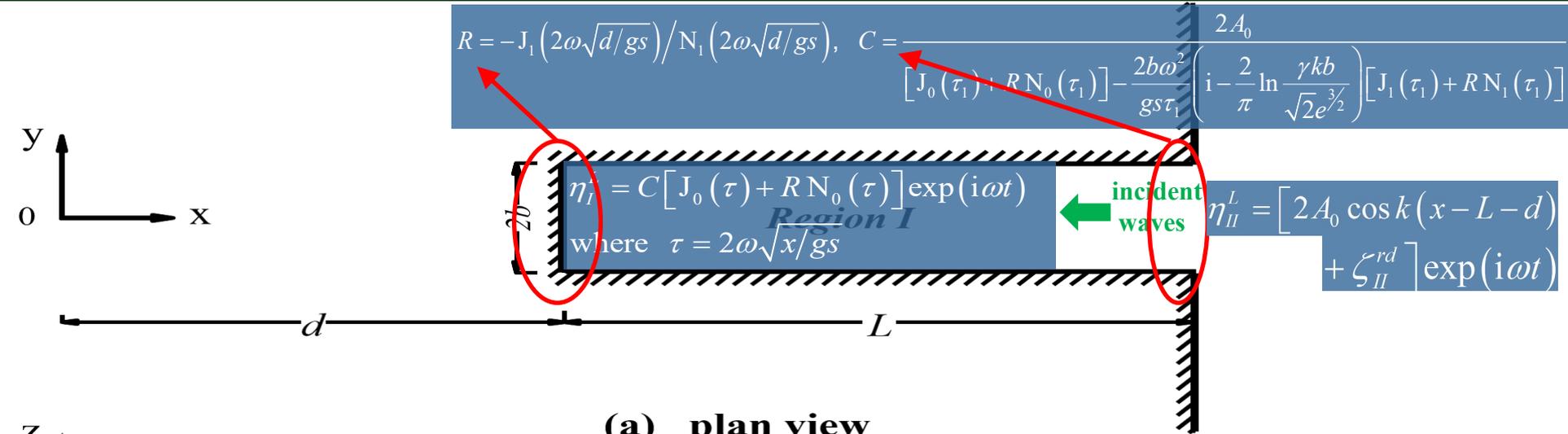
mechanisms responsible for the formation of the destructive oscillations on 15 June 2006 in Ciutadella Harbor



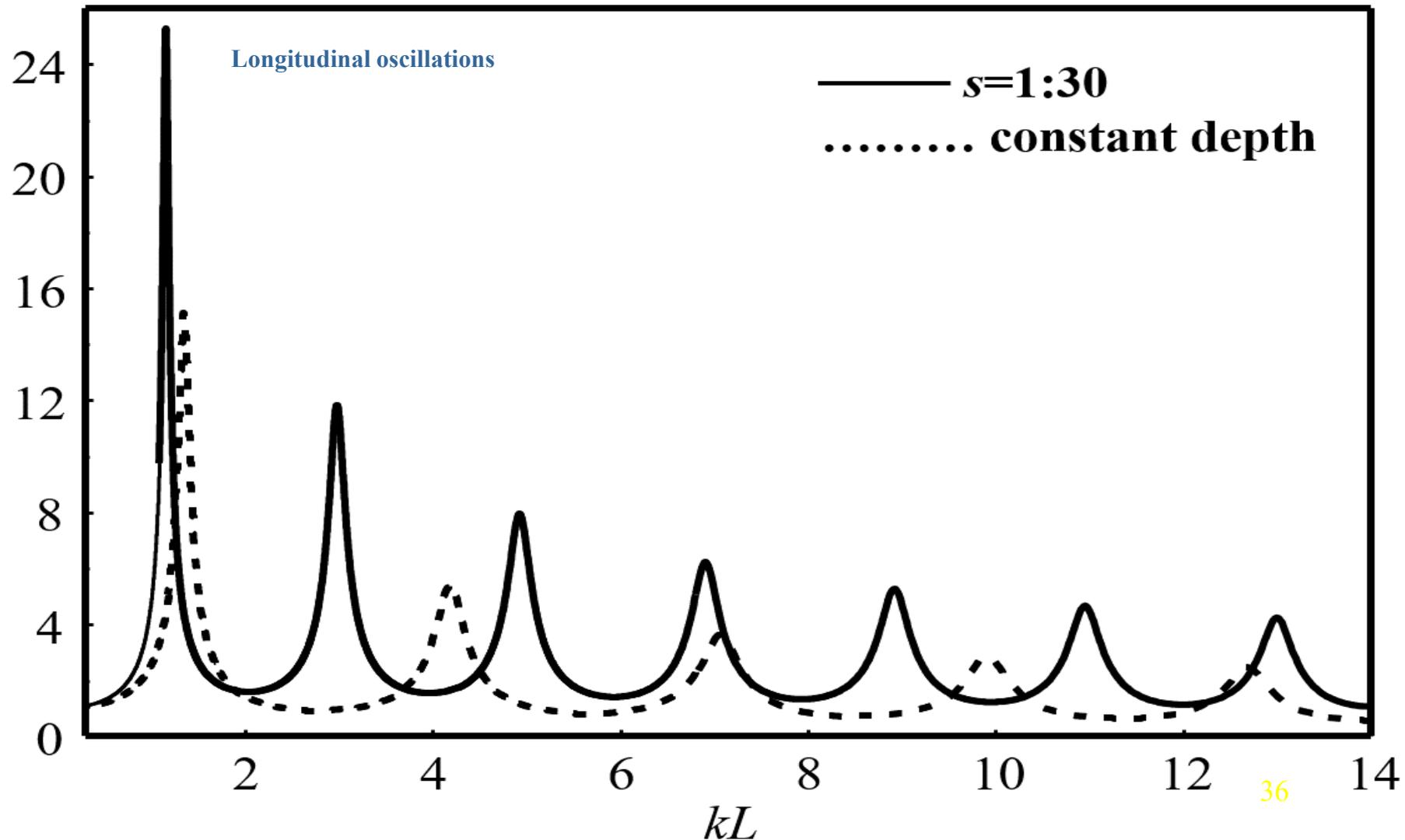
- During the meteorological tsunami of 21 June 1984, about 300 boats were destroyed or strongly damaged.
- On 15 June 2006, Ciutadella Harbor was affected by the most dramatic meteorological event of the last 20 years, when almost 6-m waves were observed in the harbor and the total economic loss was of several tens millions of Euros.

# 2 Harbor Resonance

constant slope bottom

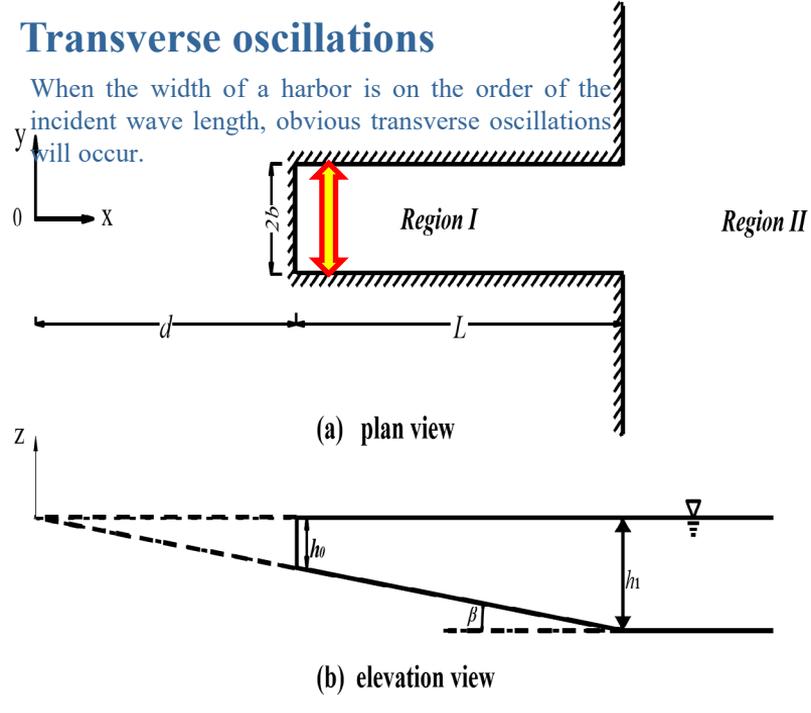


Amplification factor  $F$



## Transverse oscillations

When the width of a harbor is on the order of the incident wave length, obvious transverse oscillations will occur.



## The wavenumber

$$k_n = n\pi/2b, \quad n = 1, 2, 3, \dots$$

## The free surface

$$\eta(x, y, t) = \zeta^T(x) \sin(k_n y) \exp(i\omega t)$$

## Shallow water equations

$$x \zeta_{xx}^T + \zeta_x^T + \left( \frac{\omega^2}{gs} - k_n^2 x \right) \zeta^T = 0$$

$$\chi = 2k_n x \quad \zeta^T = \exp(-\chi/2) f(\chi)$$

## Confluent hypergeometric equation

$$\chi \cdot \frac{d^2 f}{d\chi^2} + (1 - \chi) \cdot \frac{df}{d\chi} - \alpha f = 0 \quad \text{where } \omega^2 = gsk_n(1 - 2\alpha)$$

## Free surface elevation

$$\eta^T(x, y, t) = q \exp(-k_n x) G(\alpha, 1, 2k_n x) \sin(k_n y) \exp(i\omega t)$$

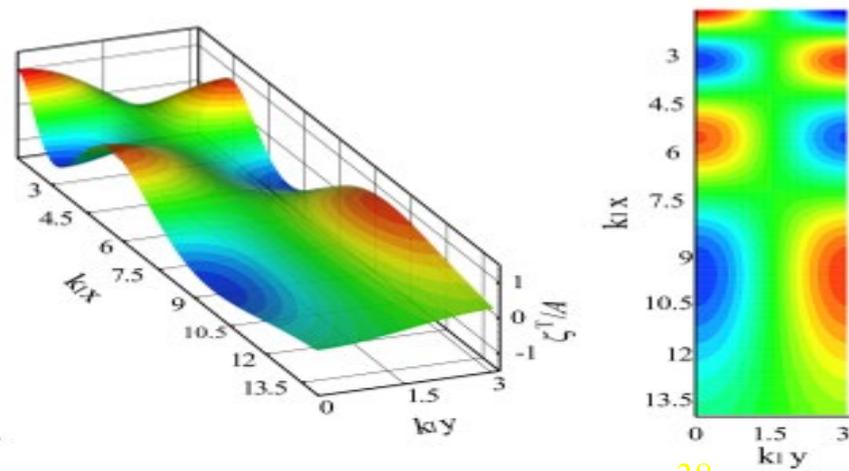
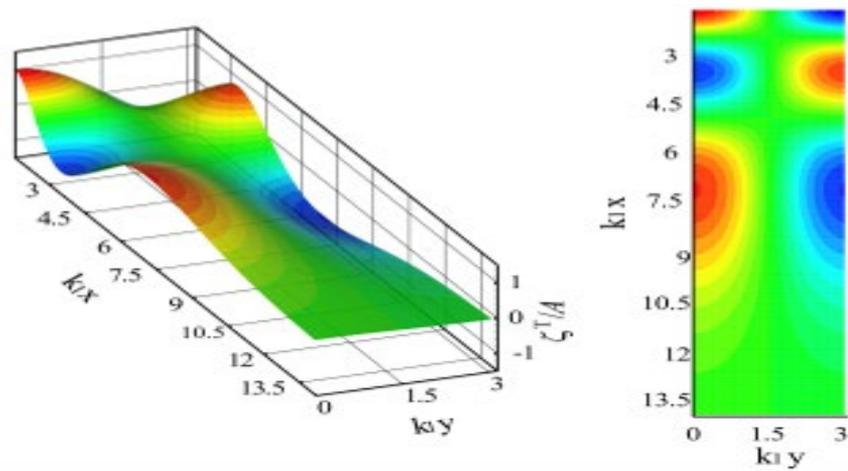
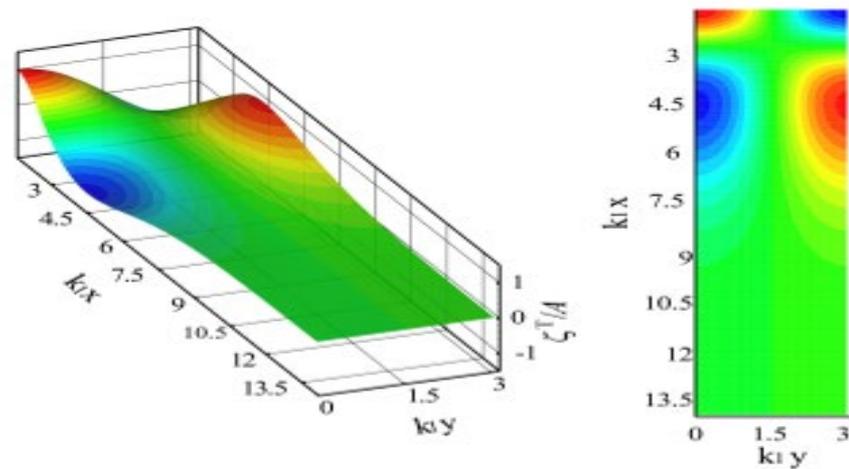
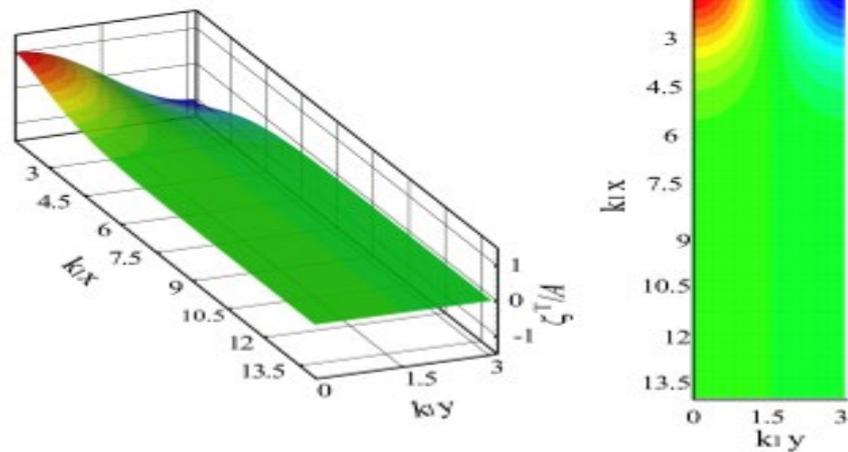
## Non-normal flux at the backwall

$$\left. \frac{\partial \eta^T}{\partial x} \right|_{x=d} = 0$$

## Dispersion relation

$$G(\alpha, 1, 2k_n d) + 2\alpha G(\alpha + 1, 2, 2k_n d) = 0$$

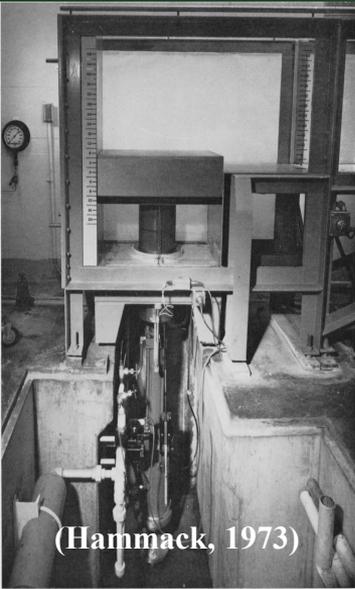
$$\omega^2 = gsk_n(1 - 2\alpha)$$



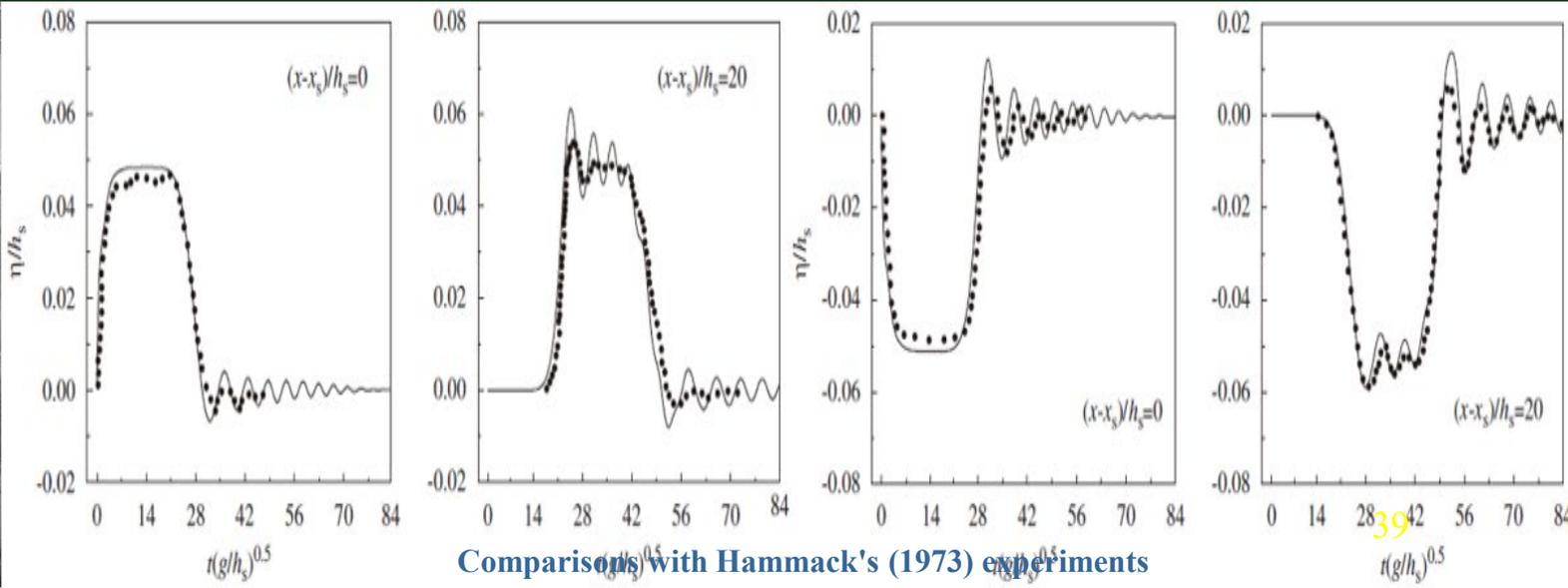
## Boussinesq equations considering seabed movements

$$\eta_t + h_t + \nabla \cdot [(h + \eta) \mathbf{u}_\alpha] + \nabla \cdot \left\{ \left( \frac{z_\alpha^2}{2} - \frac{h^2}{6} \right) h \nabla (\nabla \cdot \mathbf{u}_\alpha) + \left( z_\alpha + \frac{h}{2} \right) h \nabla [\nabla \cdot (h \mathbf{u}_\alpha) + h_t] \right\} = 0$$

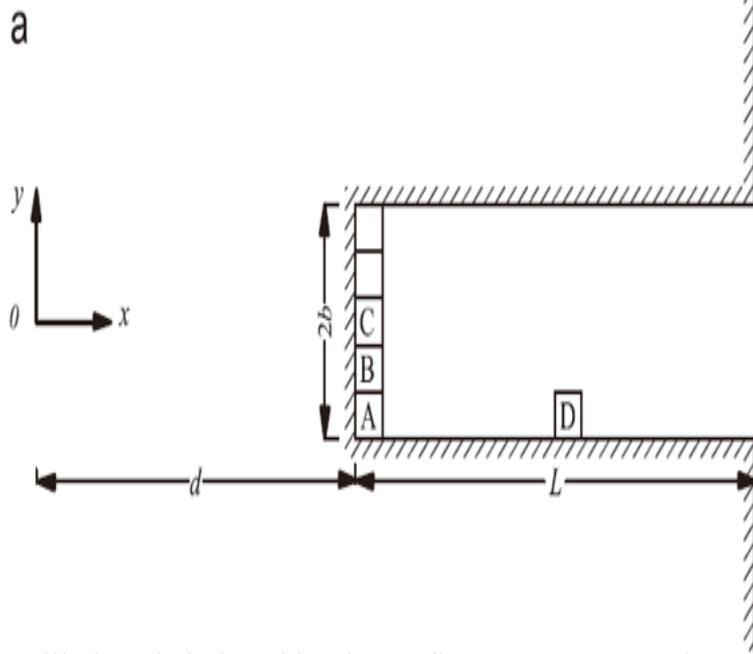
$$\mathbf{u}_{\alpha t} + \nabla \eta + (\mathbf{u}_\alpha \cdot \nabla) \mathbf{u}_\alpha + \frac{\partial}{\partial t} \left\{ \frac{z_\alpha^2}{2} \nabla (\nabla \cdot \mathbf{u}_{\alpha t}) + z_\alpha \nabla [\nabla \cdot (h \mathbf{u}_\alpha) + h_t] \right\} = 0$$



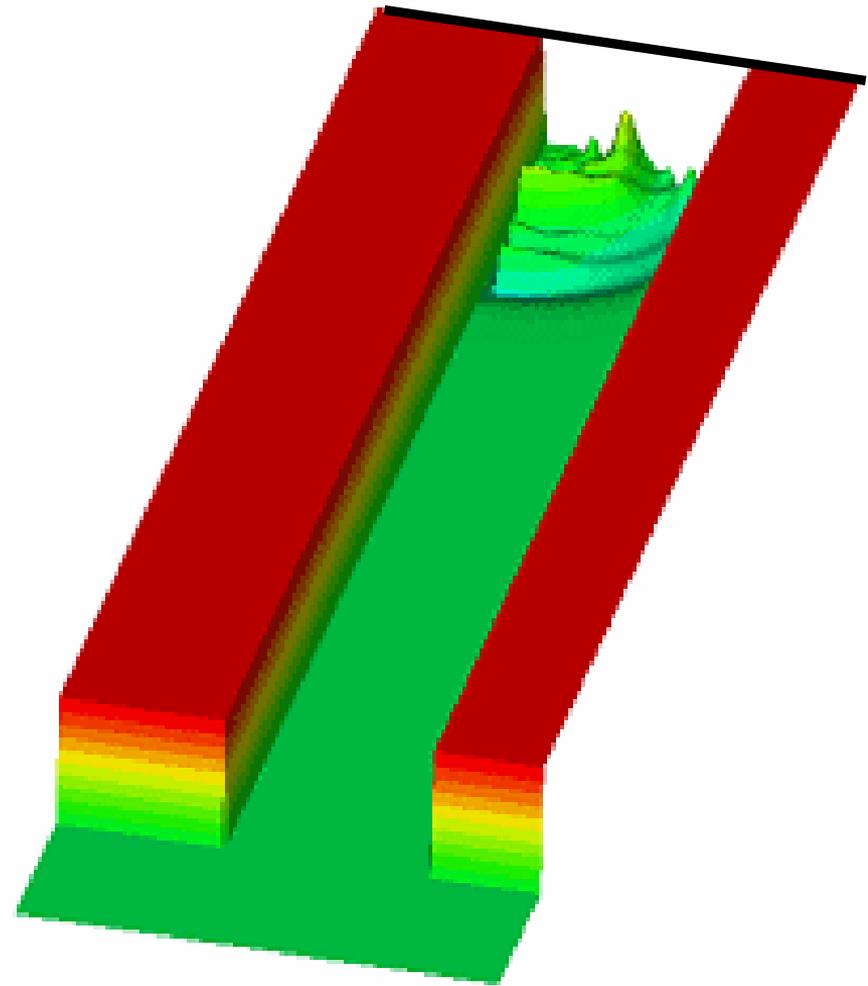
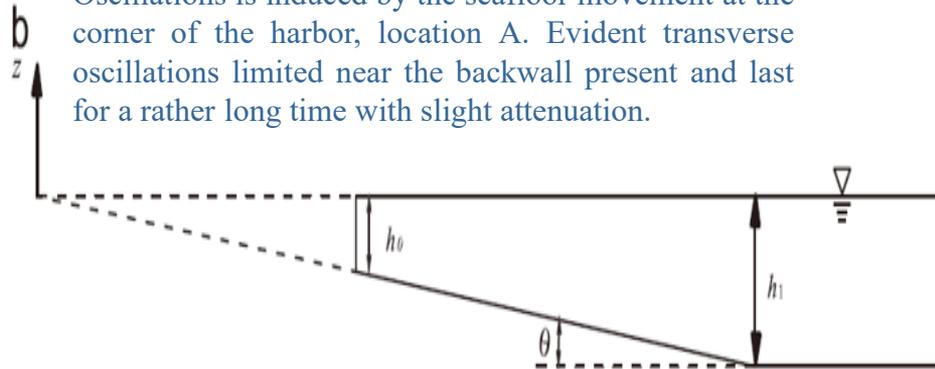
(Hammack, 1973)

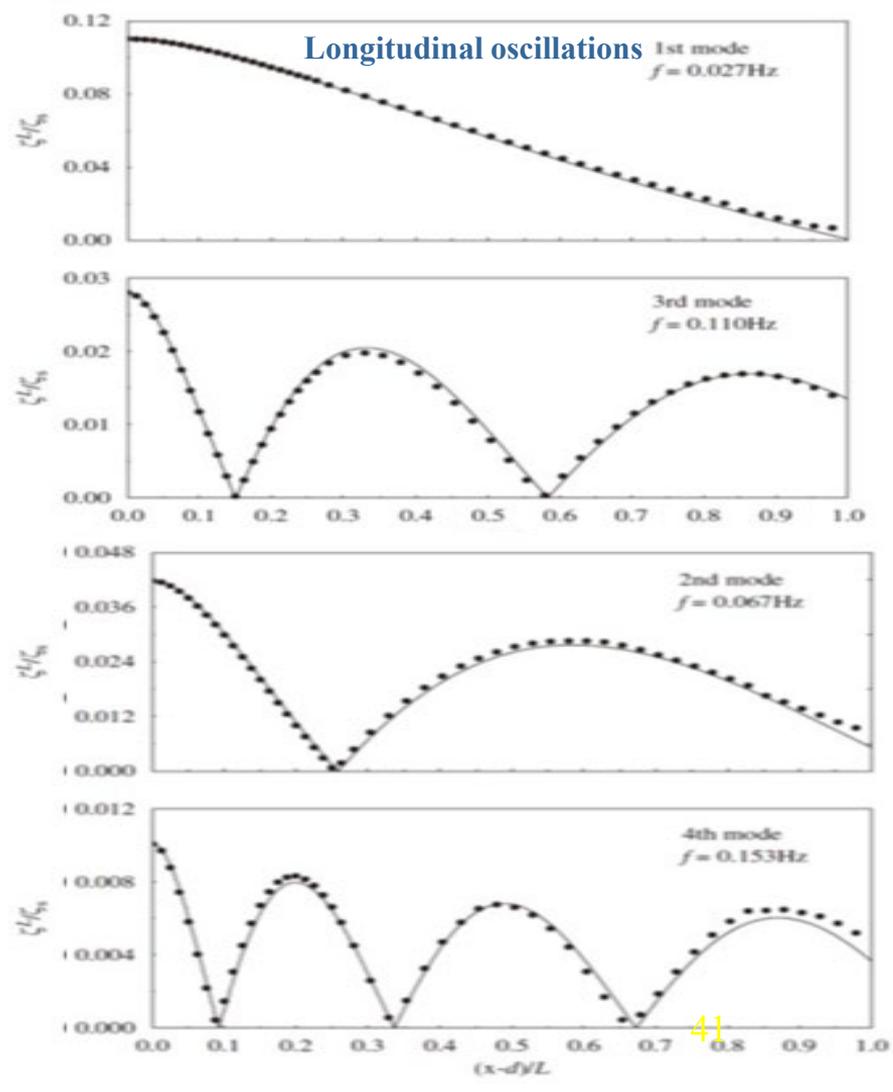
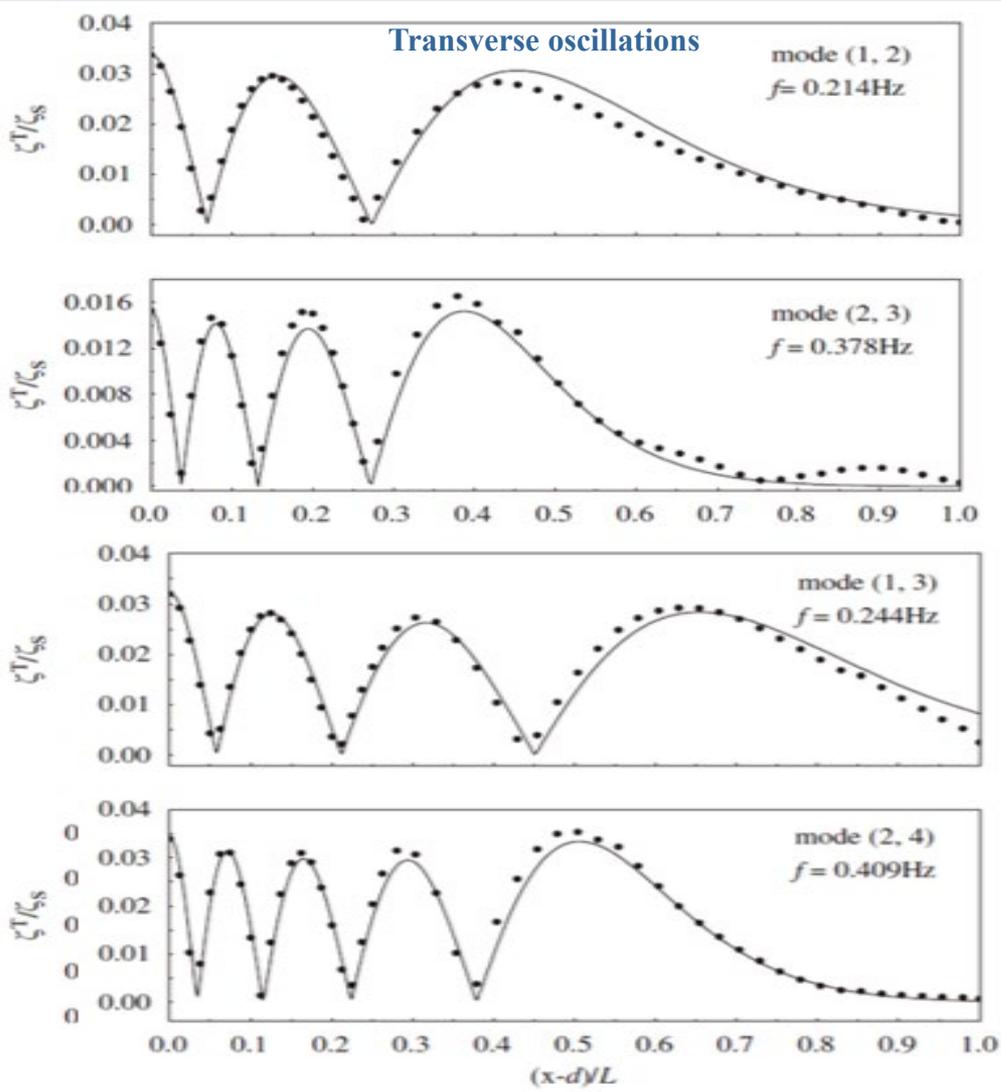


Comparisons with Hammack's (1973) experiments

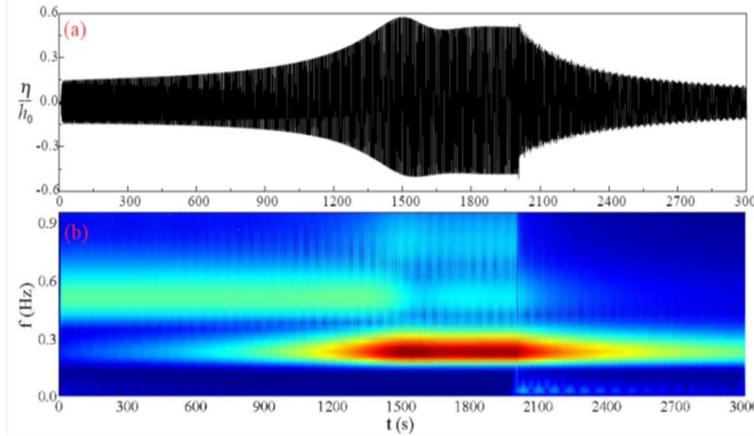
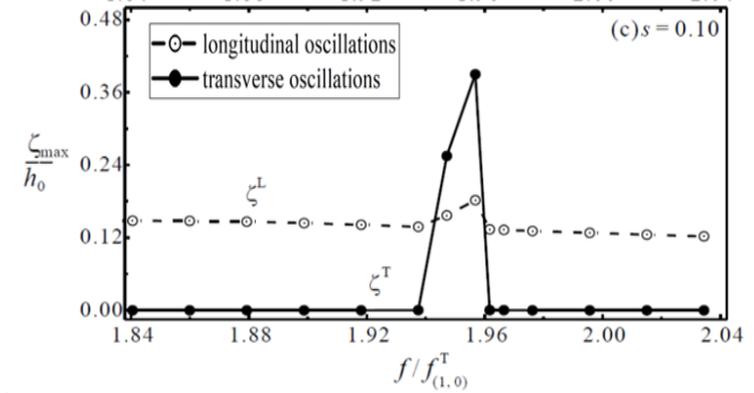
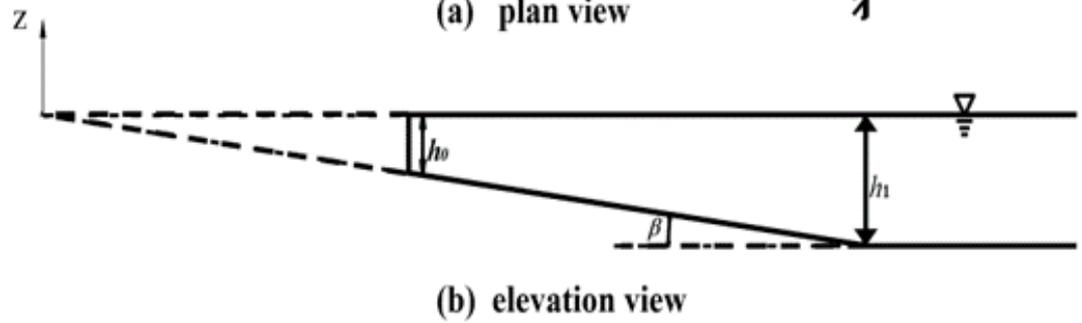
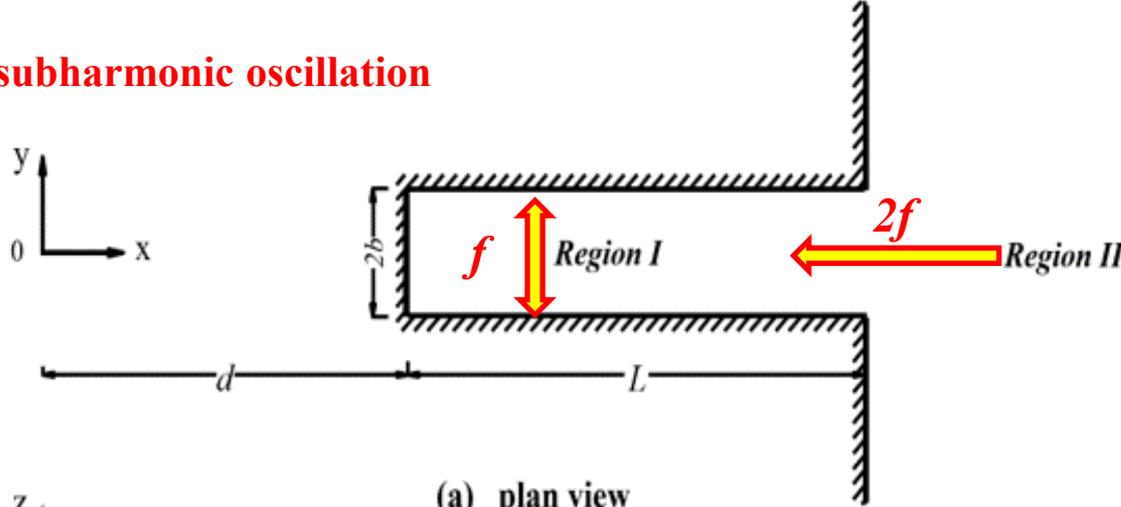


Oscillations is induced by the seafloor movement at the corner of the harbor, location A. Evident transverse oscillations limited near the backwall present and last for a rather long time with slight attenuation.



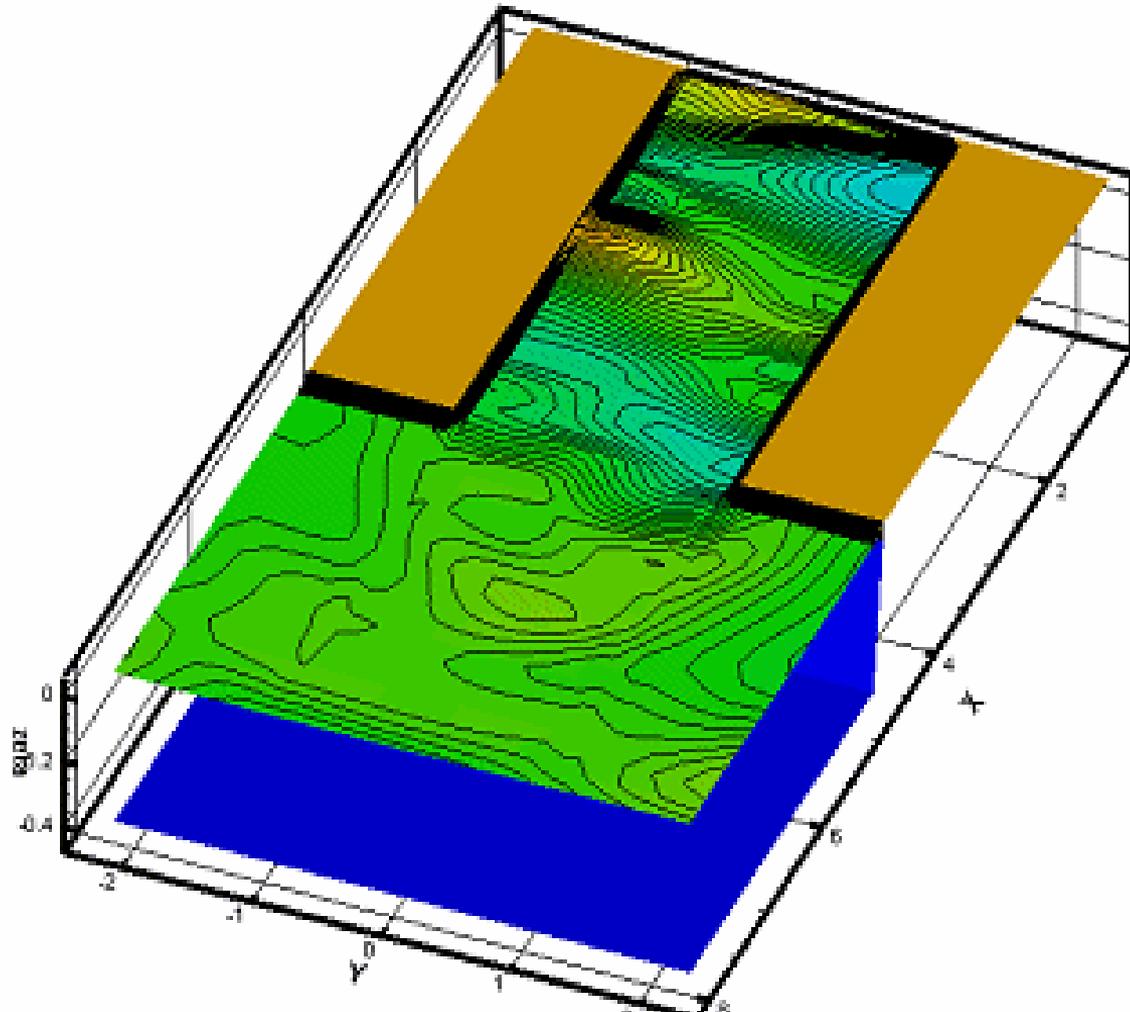


**subharmonic oscillation**



These transverse oscillations only present when the frequency of incident waves is close to the twice frequency of transverse oscillations, that is subharmonic oscillation. The wavelet spectrum show that transverse oscillations increase slowly and then approach a steady state. The magnitude of transverse oscillations is much larger than that of longitudinal oscillations.

## subharmonic oscillation



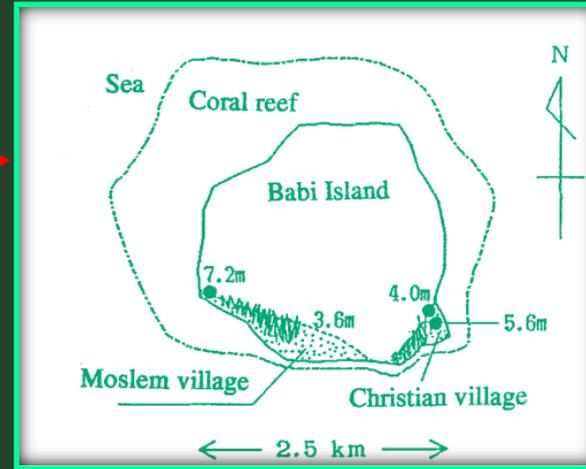
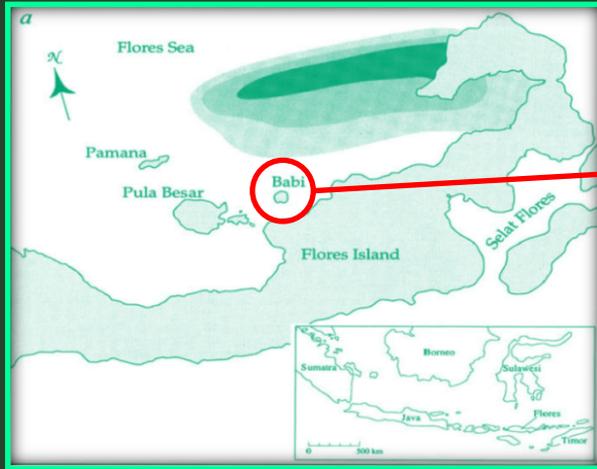
### 3 Trapped Waves around Islands



Small islands in the vicinity of the mainland are widely believed to offer protection from wind and waves and thus coastal communities have been developed in mainland areas behind small islands. whether they offer protection from long waves, such as tsunamis, is unclear.

# 3 Trapped Waves around Islands

Babi Island has a conical shape, with a diameter of approximately 2 km and a summit elevation of 351 m. The north shore faces the Flores Sea and has a wide coral reef.



(Yeh et al., 1994)

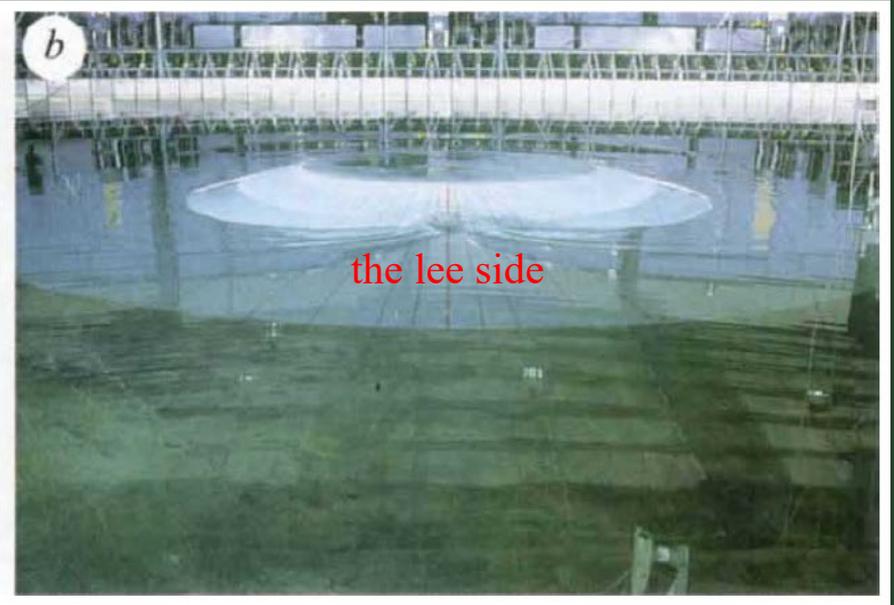
On 12 December 1992, an Mw7.5 earthquake occurred in Flores sea, Indonesia. Two villages ( Kampungbaru and Pagaraman) were destroyed, the villages were though to locate in the "shadow" zone of the wave field.

# 3 Trapped Waves around Islands



# 3 Trapped Waves around Islands

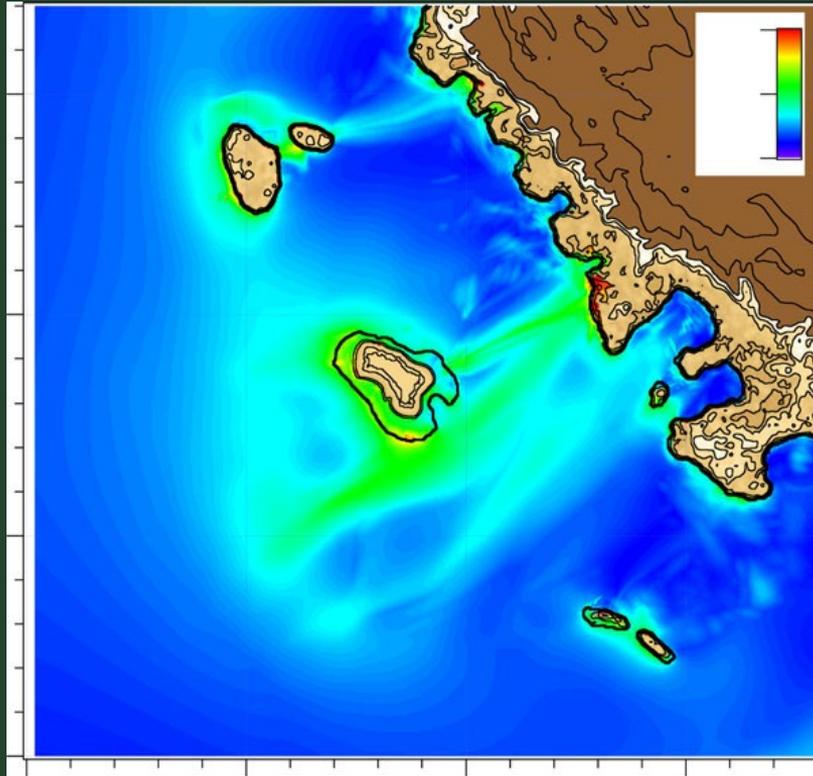
(Yeh et al., 1994)



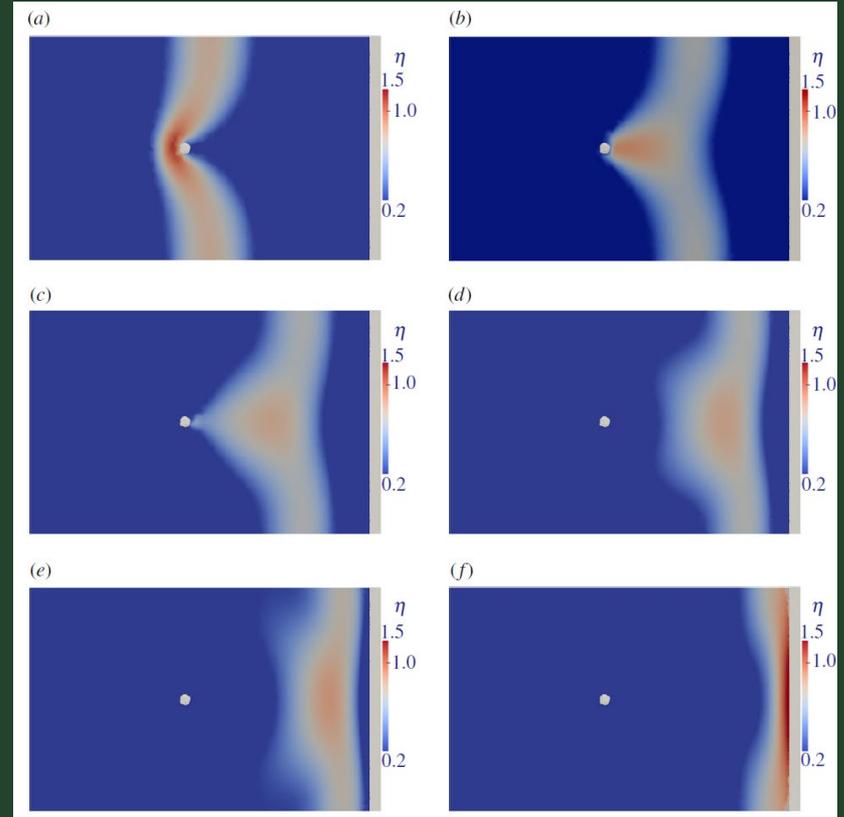
Laboratory experiments were conducted.

The tsunami wave was split into two, with one wave propagating around each side of the island. The two waves met in the sheltered region, and the subsequent amplification of wave amplitude resulted in the destructive flow onto the beach.

# 3 Trapped Waves around Islands



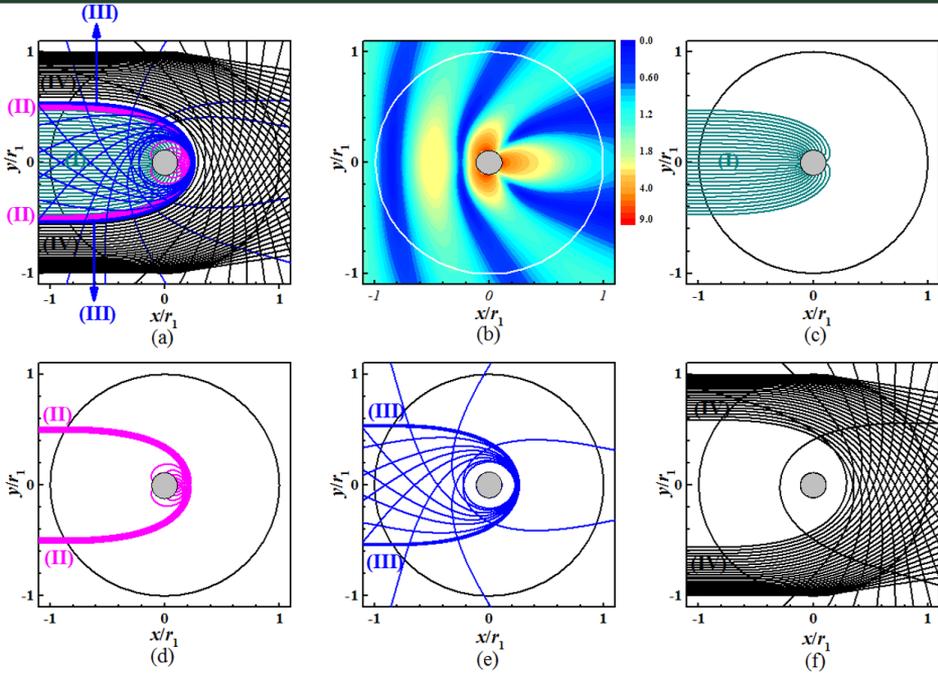
The maximum wave height during 25 October 2010 Mentawai tsunami



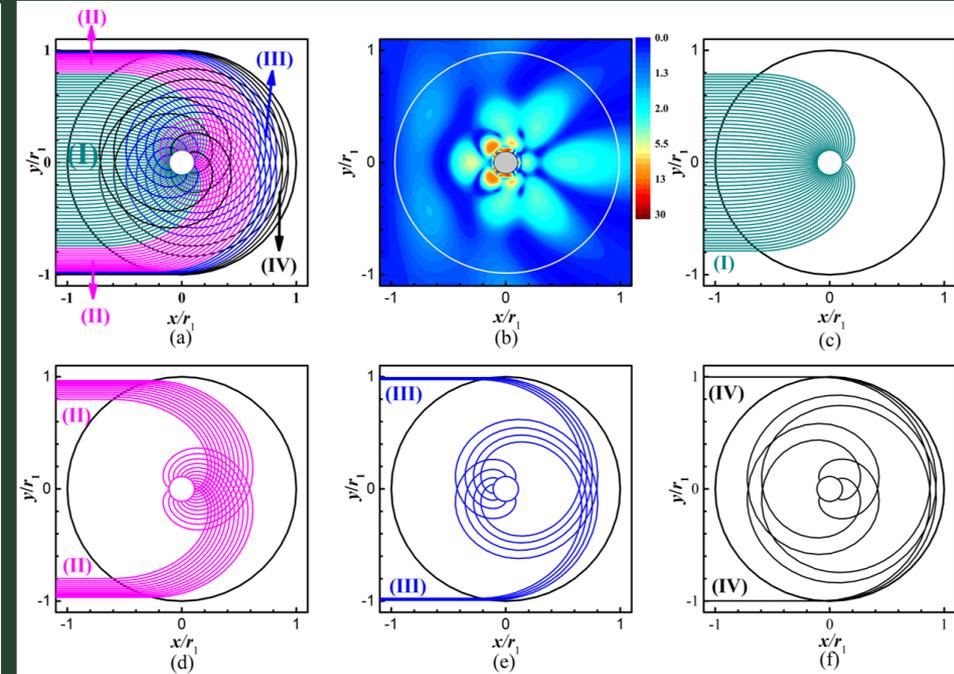
Snapshots of the free surface elevation when the wave passes the island and runs up the beach behind it. <sup>48</sup>

# 3 Trapped Waves around Islands

gentle slope

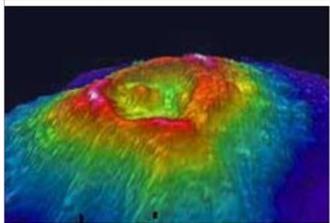


steep slope

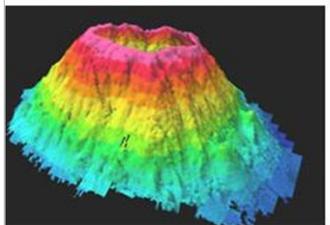


Wave transformations are high related to the topography.

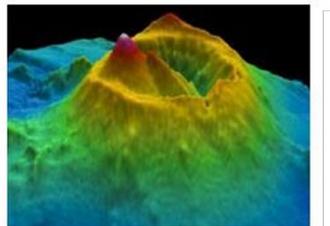
# 3 Trapped Waves around Islands



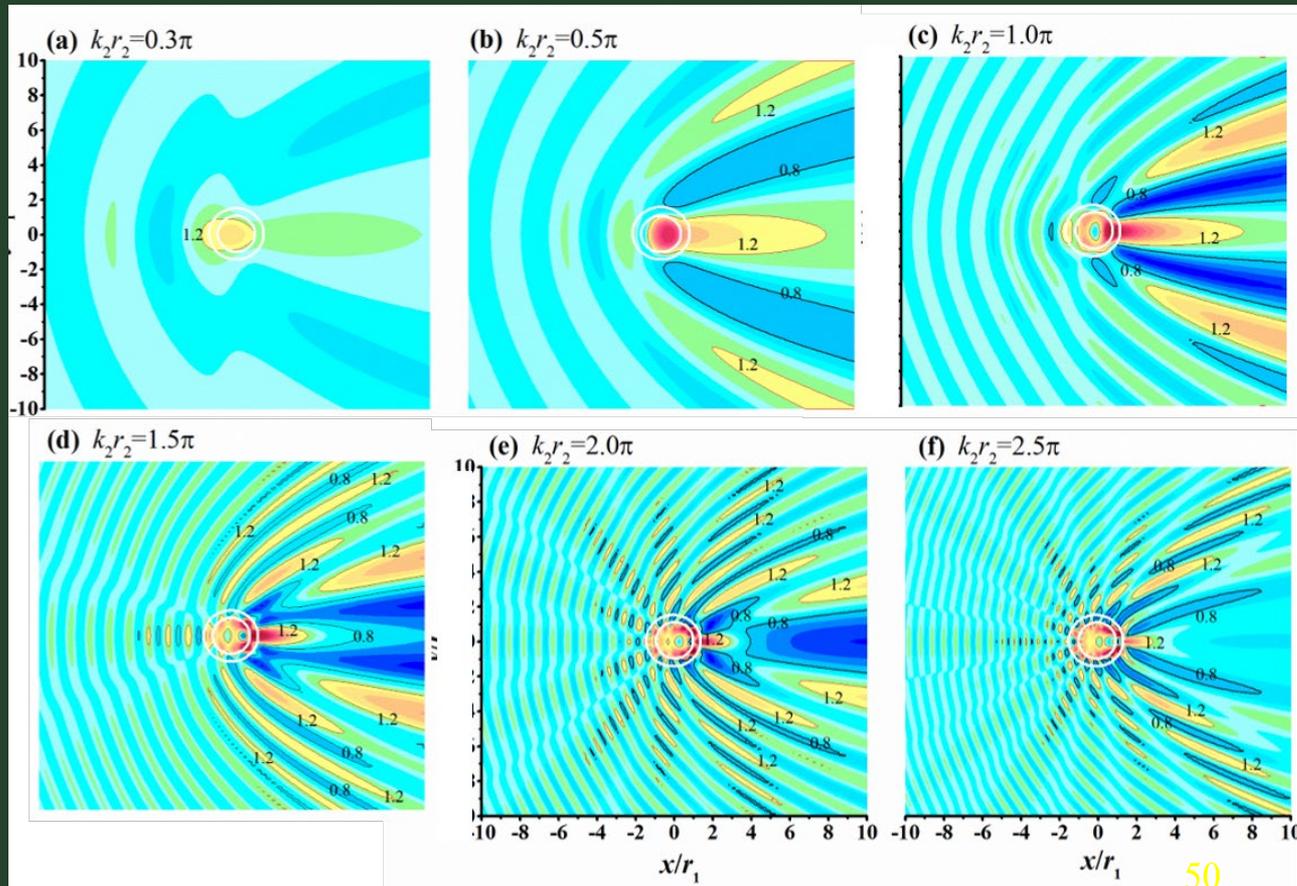
Vailulu'u 海底山



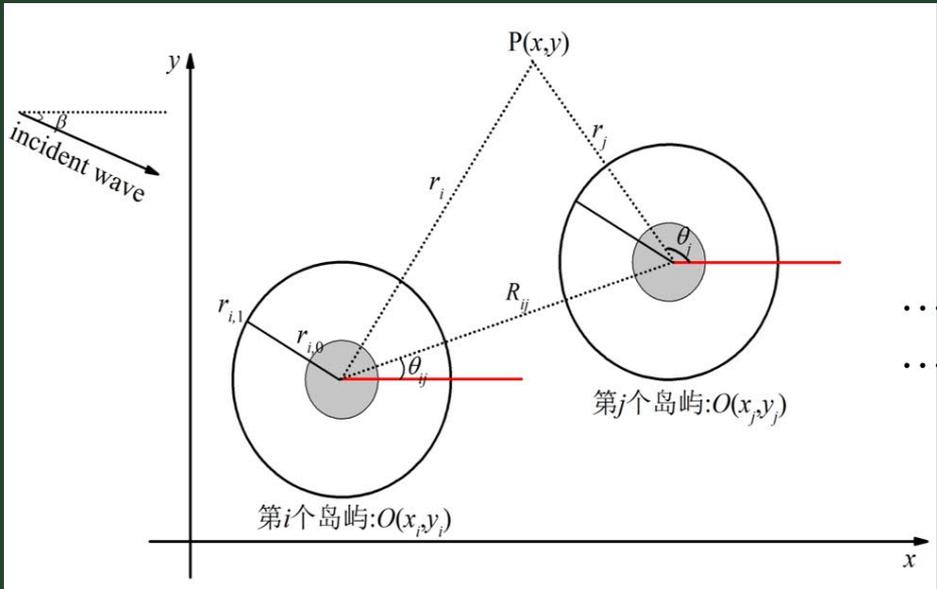
Vema 海底山



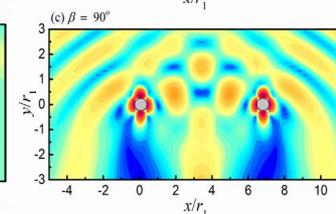
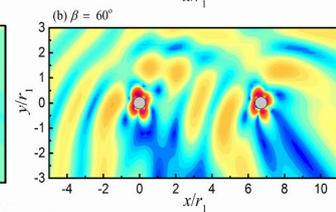
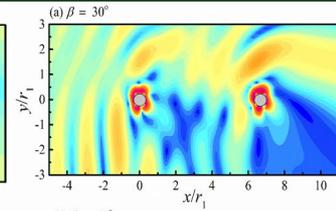
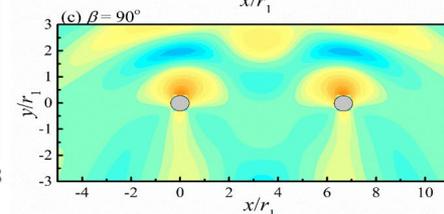
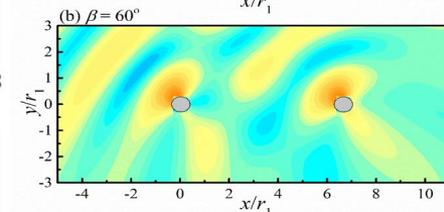
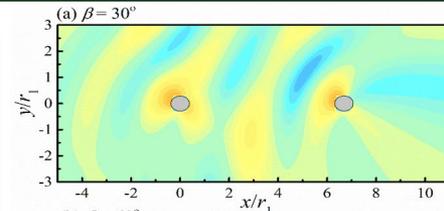
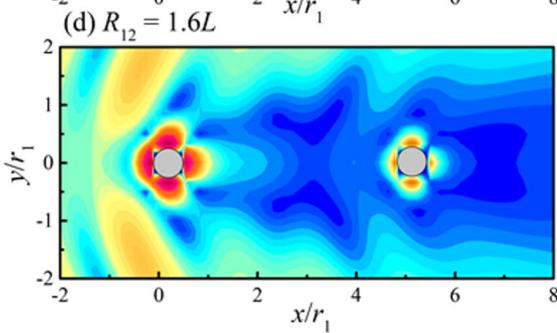
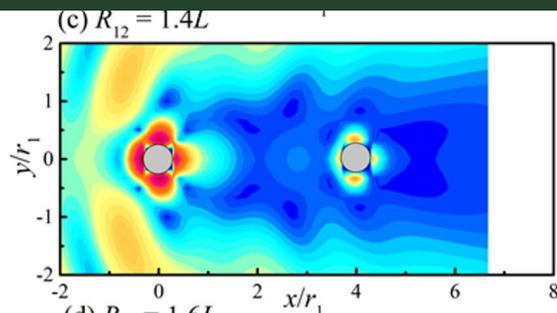
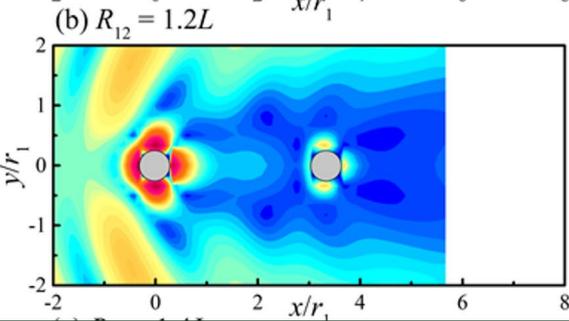
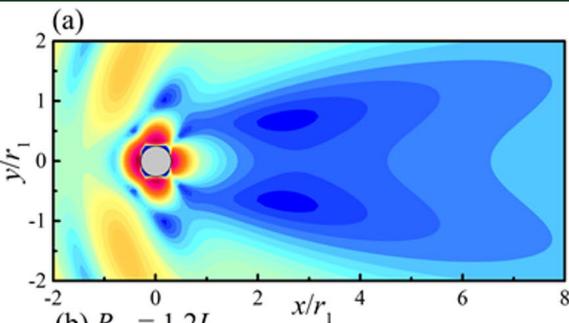
Brothers 海底山  
seamount



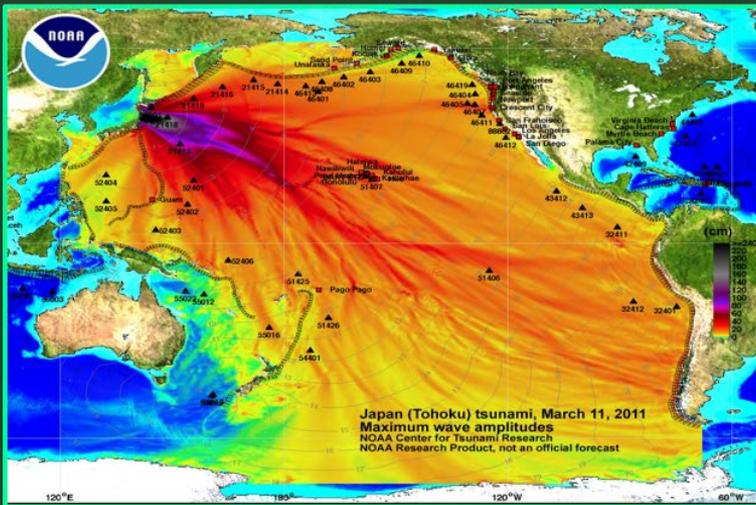
# 3 Trapped Waves around Islands



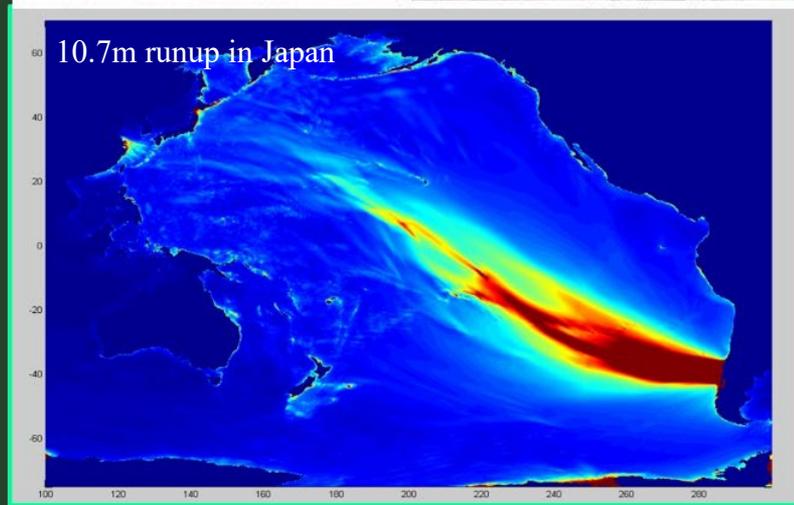
# 3 Trapped Waves around Islands



# 4 Trapped Waves over Ridges



2011 Tohoku Tsunami

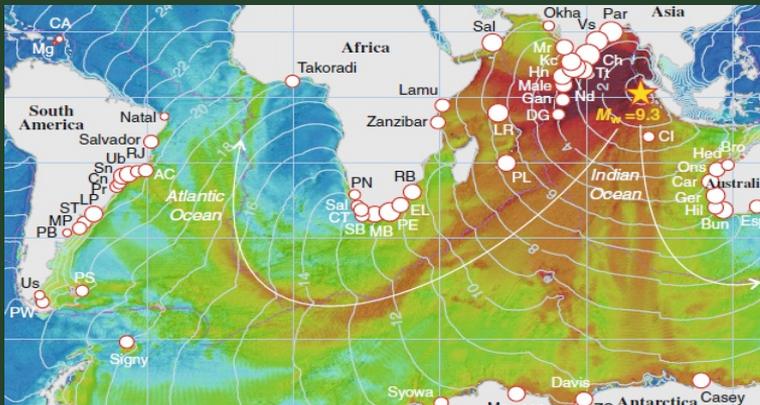


1960 Great Chile Tsunami

# 4 Trapped Waves over Ridges



JOURNAL ARTICLE  
The Global Reach of the 26 December  
2004 Sumatra Tsunami

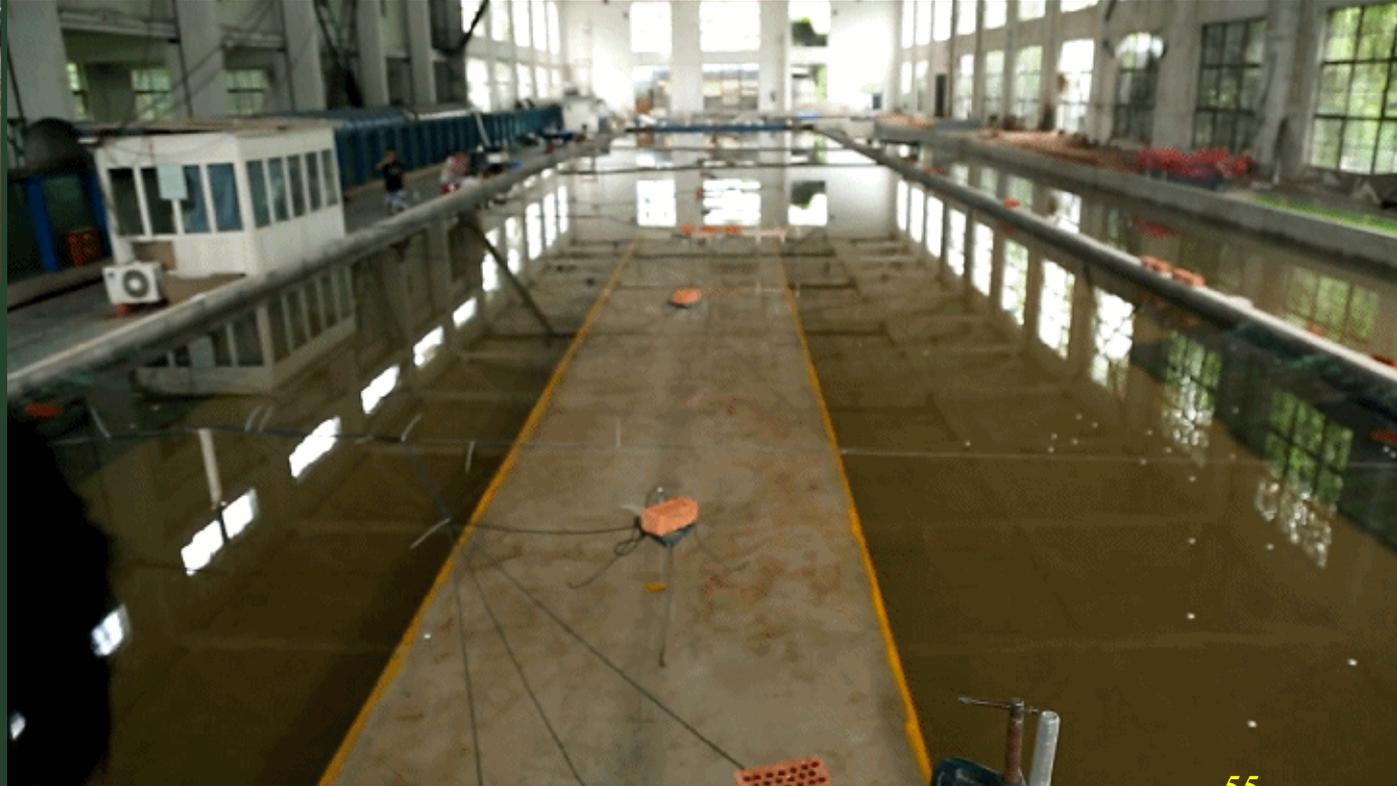


		Tsunami in the Southern America		
		Travel time (h:min)	Maximum wave	
			Observ. time (UTC)	Height (cm)
<i>Juan Fernandez Island (Chile)</i>	2	00:47 <sup>a</sup>	12:02 <sup>a</sup>	39
<i>San Felix Island (Chile)</i>	2	01:57 <sup>a</sup>	16:04 <sup>a</sup>	8
Corral (Chile)	2	00:54 <sup>a</sup>	07:34 <sup>a</sup>	10
Talcahuano (Chile)	2	01:38 <sup>a</sup>	15:20 <sup>a</sup>	29
San Antonio (Chile)	2	00:30 <sup>a</sup>	01:08 <sup>b</sup>	42
Valparaiso (Chile)	2	01:00 <sup>a</sup>	03:38 <sup>b</sup>	15
Coquimbo (Chile)	2	02:02 <sup>a</sup>	14:54 <sup>a</sup>	18
Caldera (Chile)	2	02:12 <sup>a</sup>	16:02 <sup>a</sup>	35
Antofagasta (Chile)	2	03:32 <sup>a</sup> (10:26 <sup>a</sup> )	20:56 <sup>a</sup>	22
Iquique (Chile)	2	03:44 <sup>a</sup>	12:40	26
Arica (Chile)	2	04:13 <sup>a</sup> (13:20 <sup>a</sup> )	05:12 <sup>b</sup>	24
Callao (Peru)	2	05:46 <sup>a</sup>	19:04 <sup>a</sup>	72
<i>Baltra, Galapagos Is. (Ecuador)</i>	2	06:06 <sup>a</sup>	16:08 <sup>a</sup>	67
			14:16 <sup>a</sup>	35

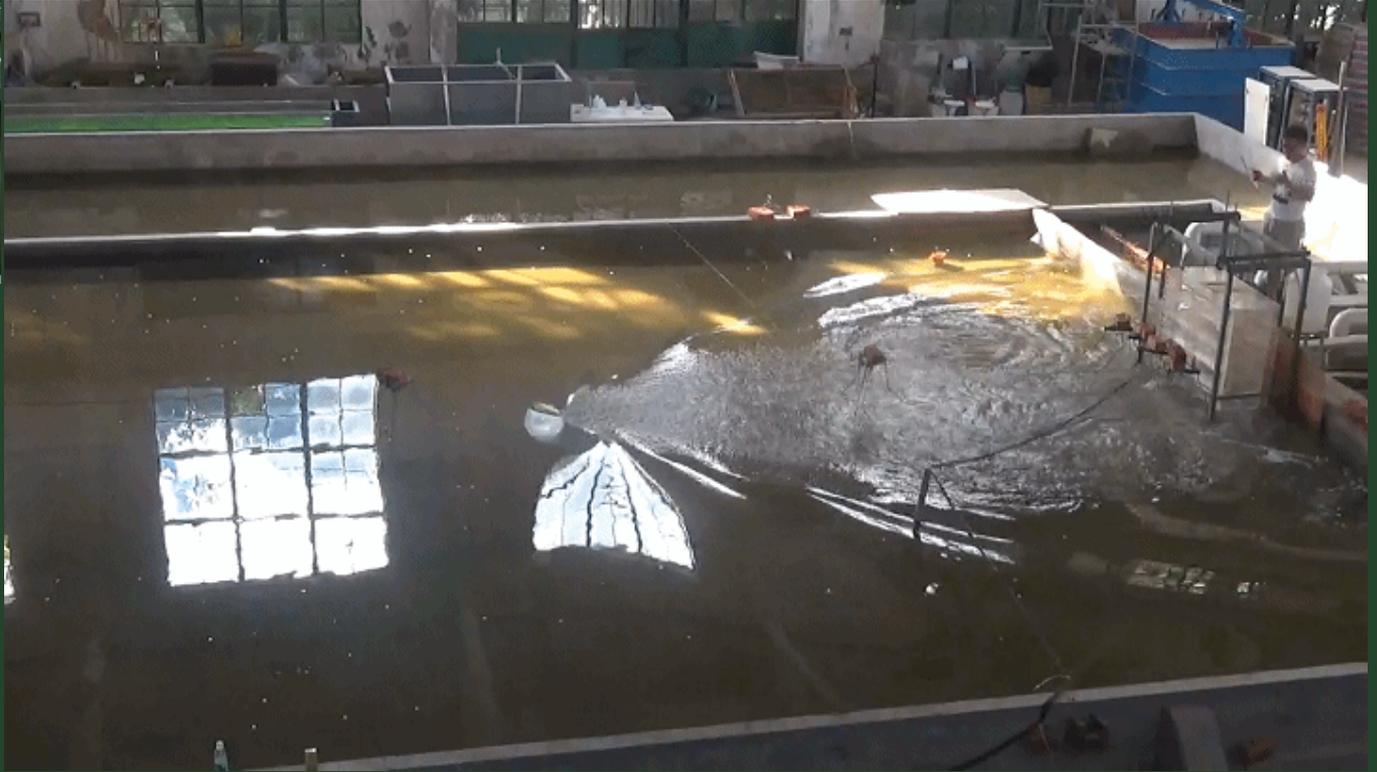
10 ~ 20 hrs delay  
tsunami coda

10. ...  
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5), pp.

# 4 Trapped Waves over Ridges



# 4 Trapped Waves over Ridges

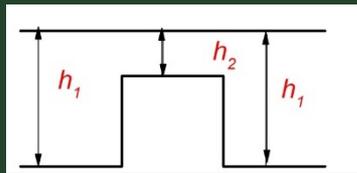


# 4 Trapped Waves over Ridges

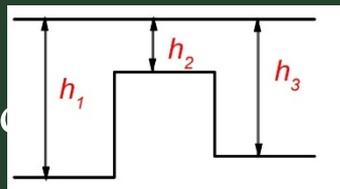


# 4 Trapped Waves over Ridges

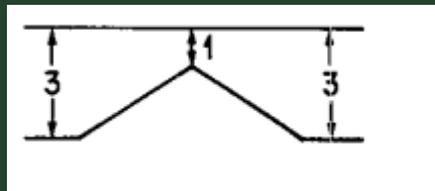
1953 Jones



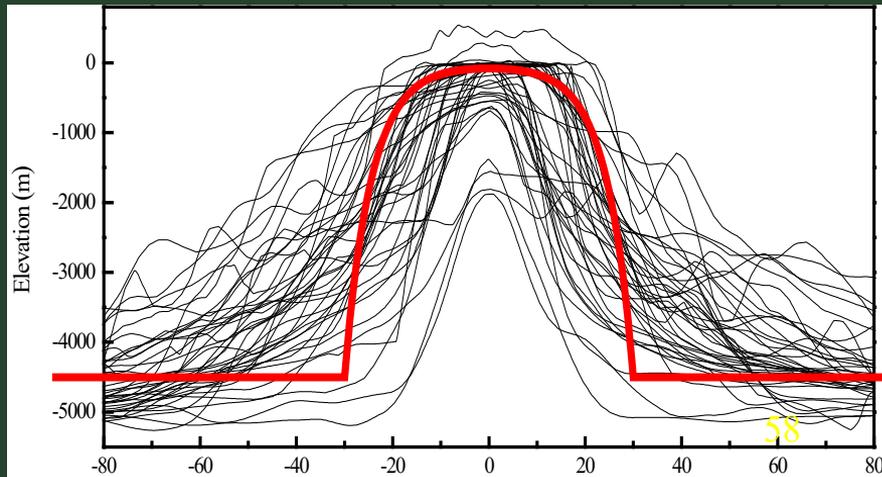
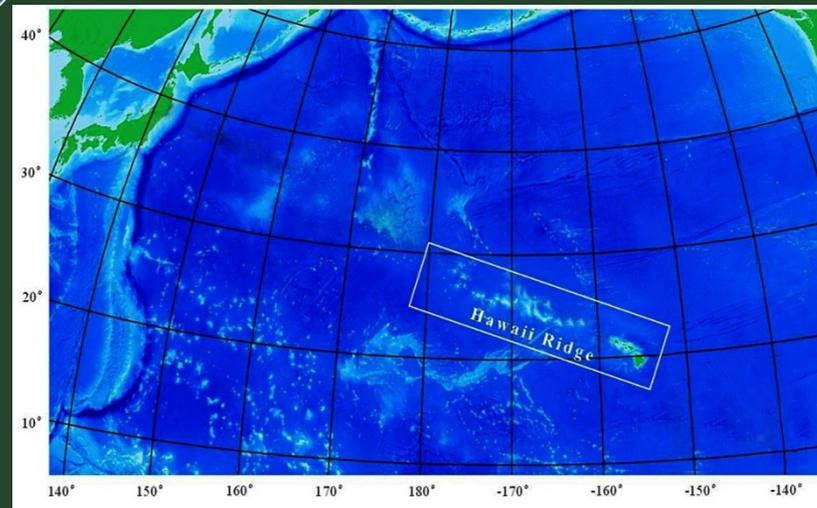
1968 Longuet-Higgins



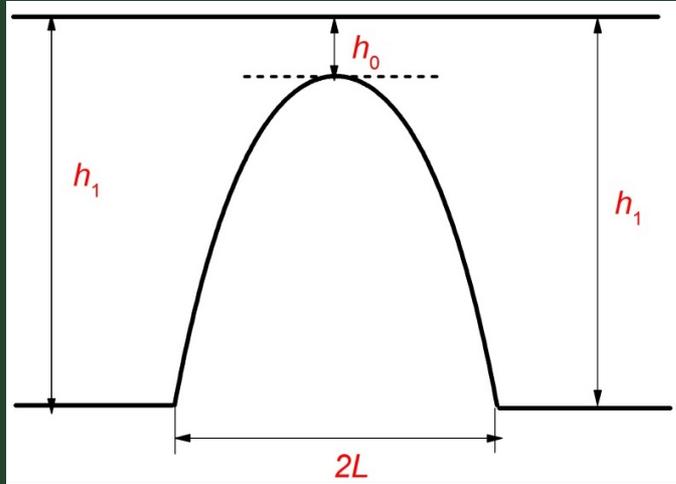
1969 Buchwald



1981 Shaw

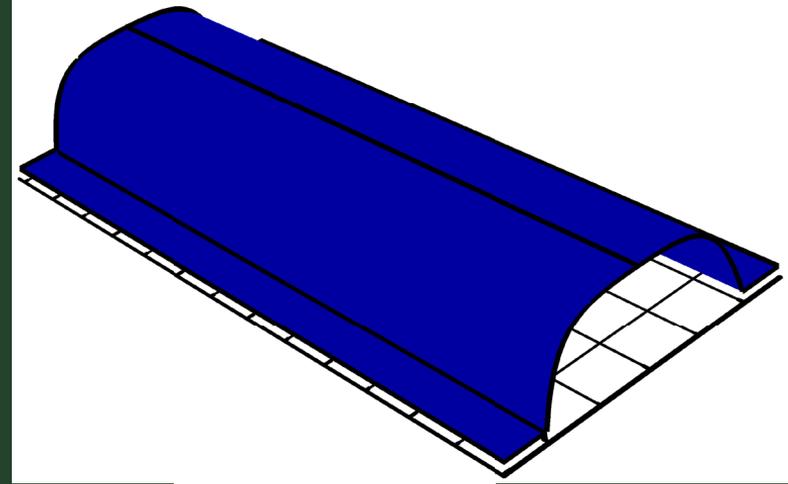


# 4 Trapped Waves over Ridges



hyperbolic-cosine ocean ridge

$$h(x) = h_0 \cosh^2(\lambda x)$$



$$\eta_{tt} - g \nabla (h \nabla \eta) = 0$$

## 4 Trapped Waves over Ridges

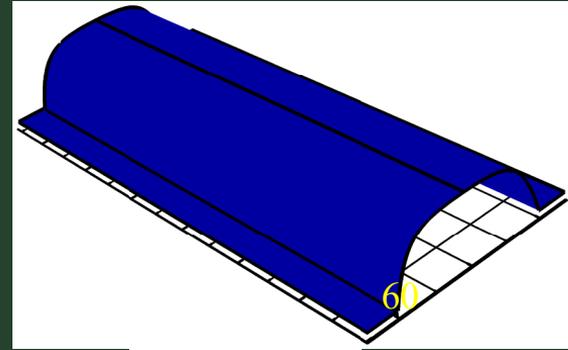
$$\eta(x, y, t) = A \operatorname{sech}^2(\lambda x) \left[ P(v, u, \chi) - \frac{2}{\pi} \tan(u\pi) Q(v, u, \chi) \right] \exp \left[ i(k_y y - \omega t) \right]$$

$$\omega^2 = gh_0 \lambda^2 \left( \sqrt{1 + \frac{k_y^2}{\lambda^2}} + 2m \right) \left( \sqrt{1 + \frac{k_y^2}{\lambda^2}} + 2m + 1 \right) \quad m = 0, 1, 2, 3 \dots$$

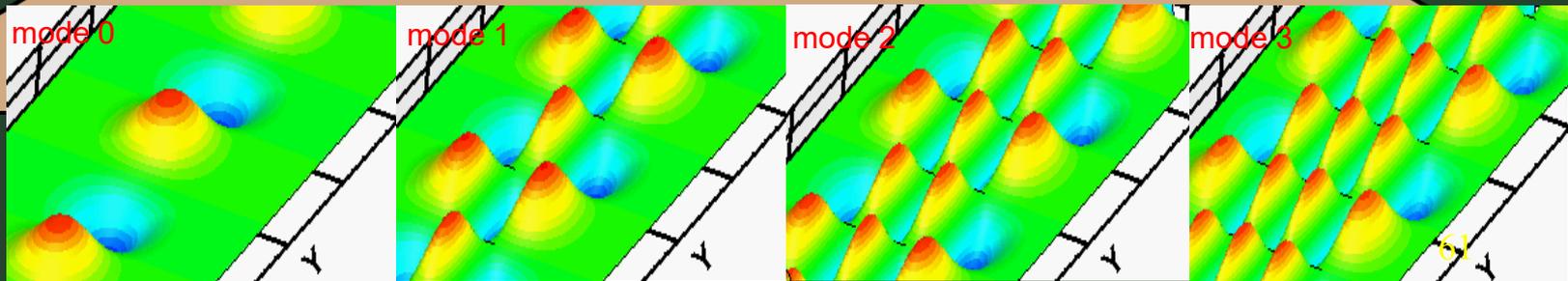
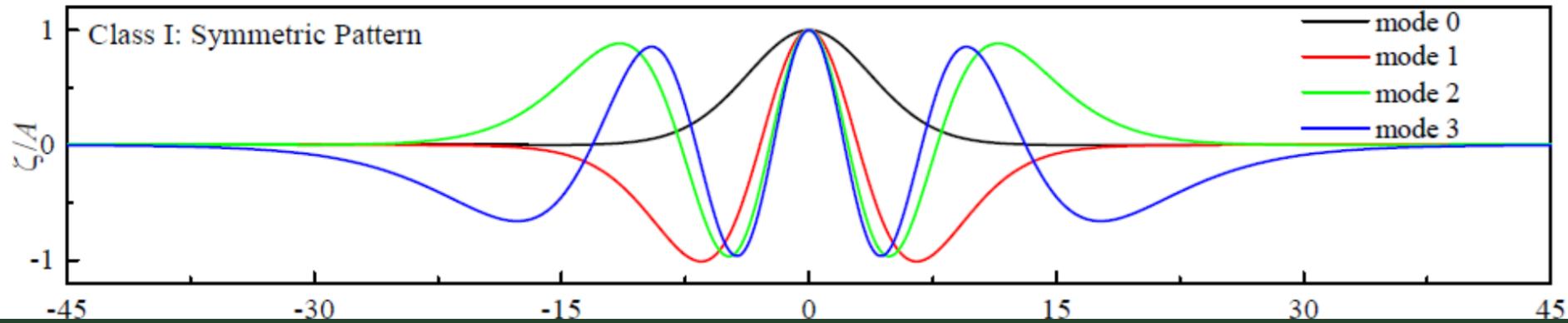
**symmetrical mode**

$$\omega^2 = gh_0 \lambda^2 \left( \sqrt{1 + \frac{k_y^2}{\lambda^2}} + 2m \right) \left( \sqrt{1 + \frac{k_y^2}{\lambda^2}} + 2m - 1 \right) \quad m = 1, 2, 3 \dots$$

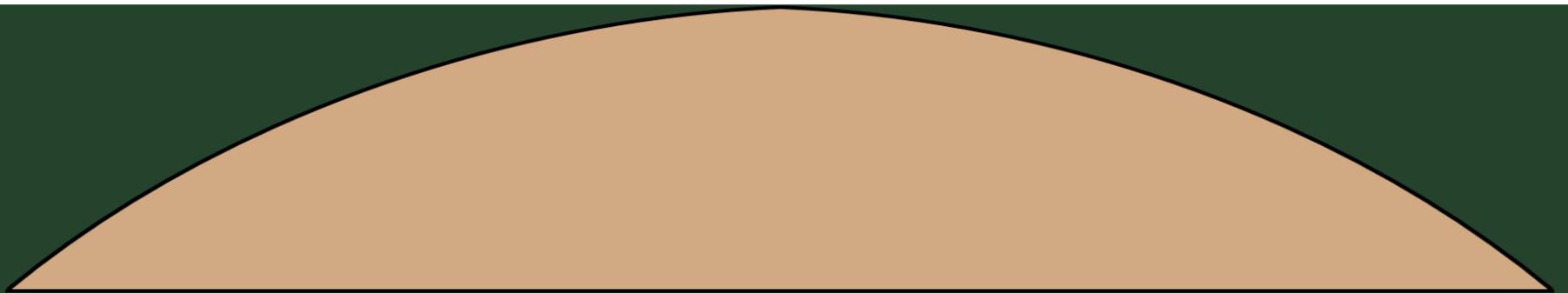
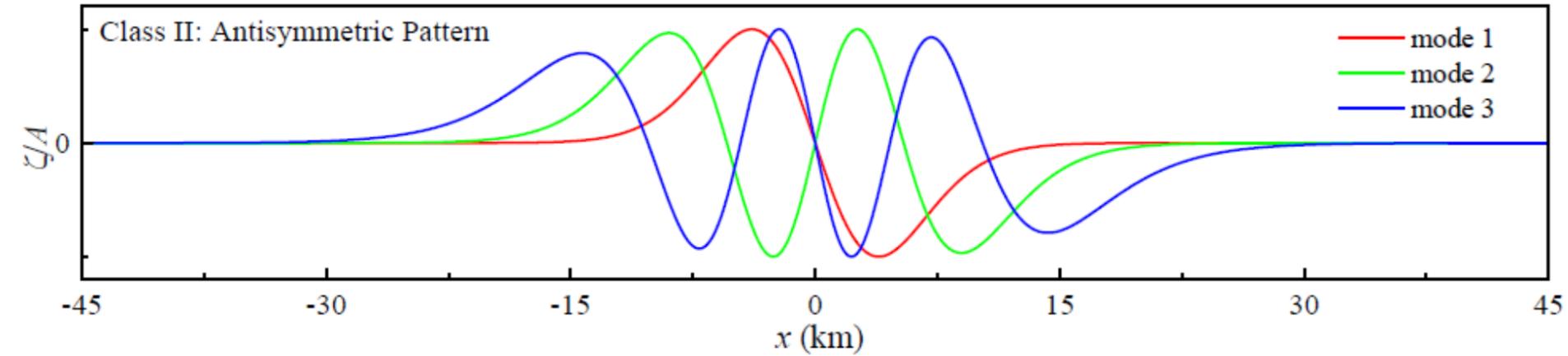
**Antisymmetrical mode**



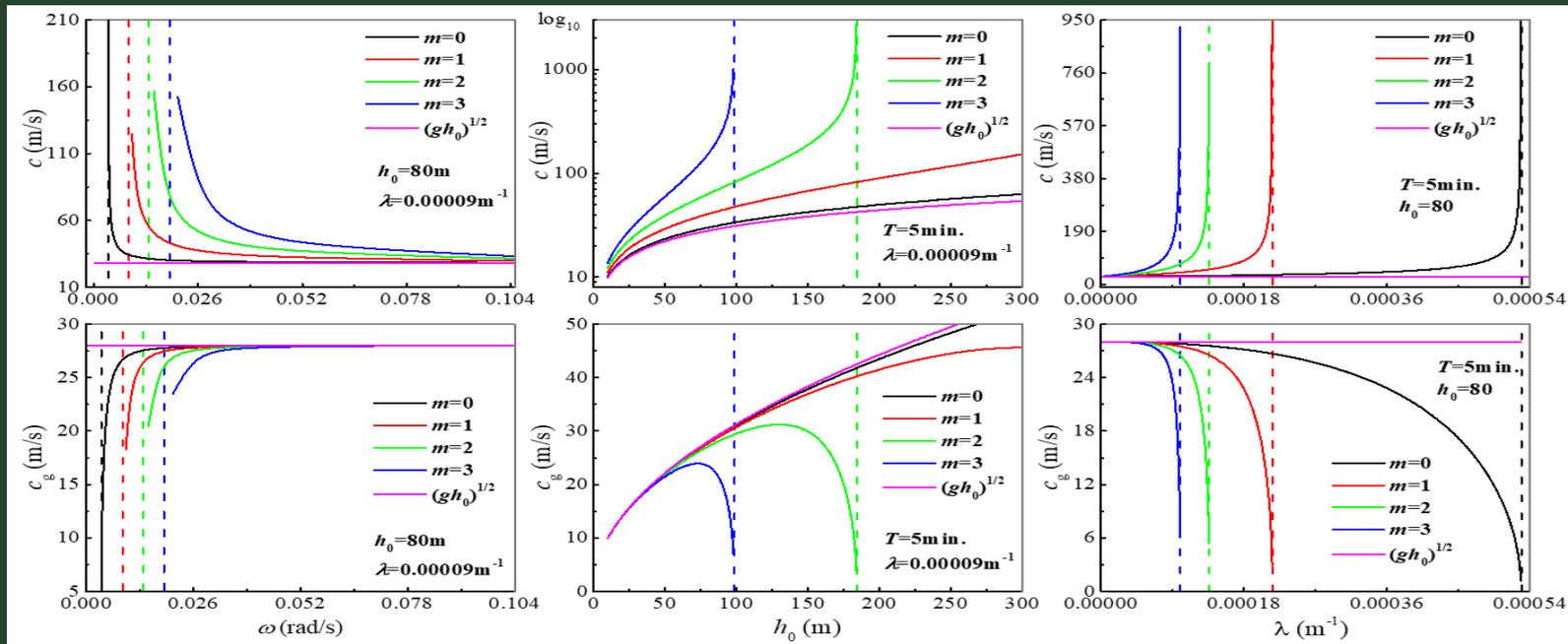
# 4 Trapped Waves over Ridges



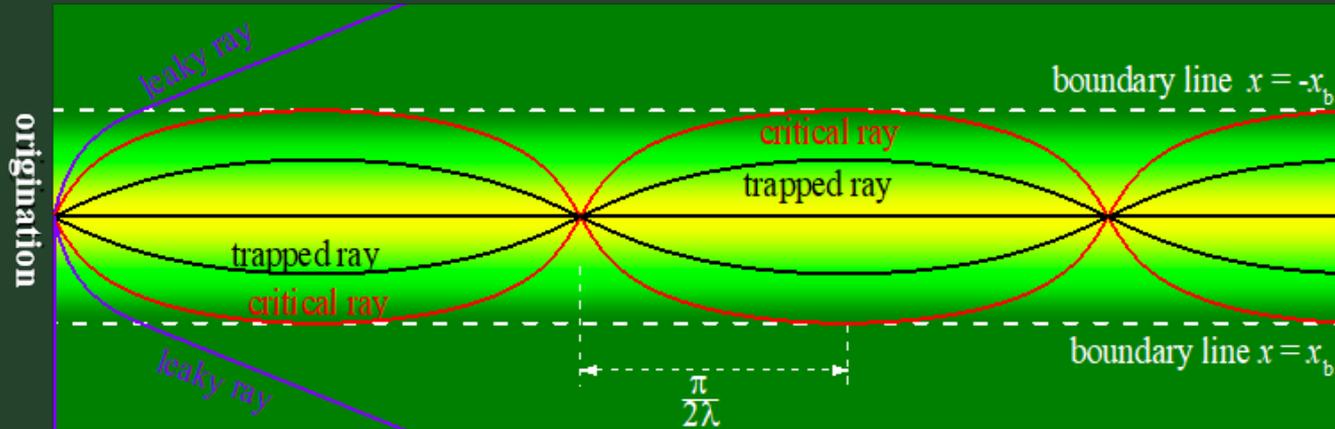
# 4 Trapped Waves over Ridges



# 4 Trapped Waves over Ridges

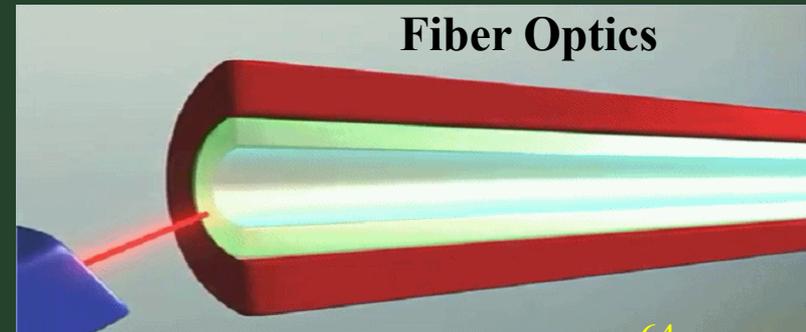


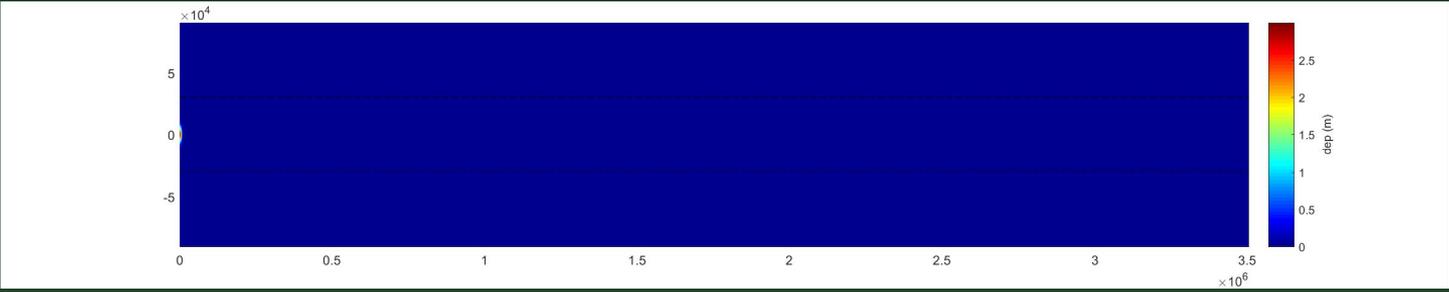
# 4 Trapped Waves over Ridges

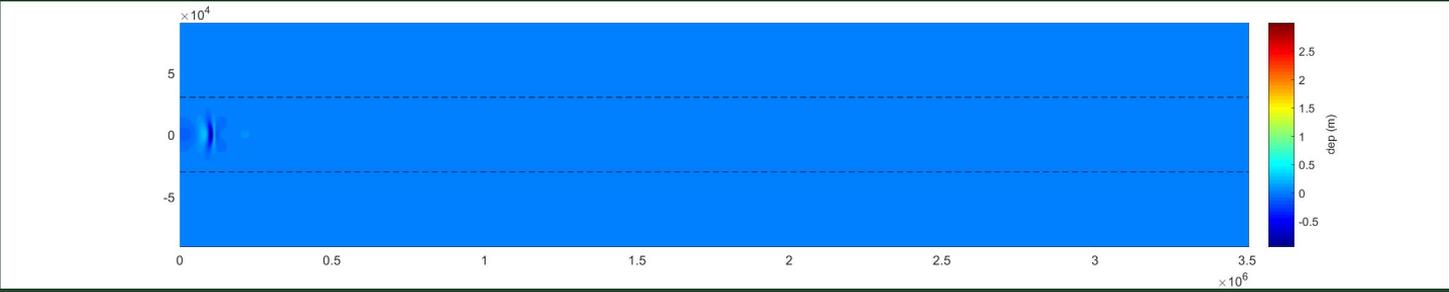


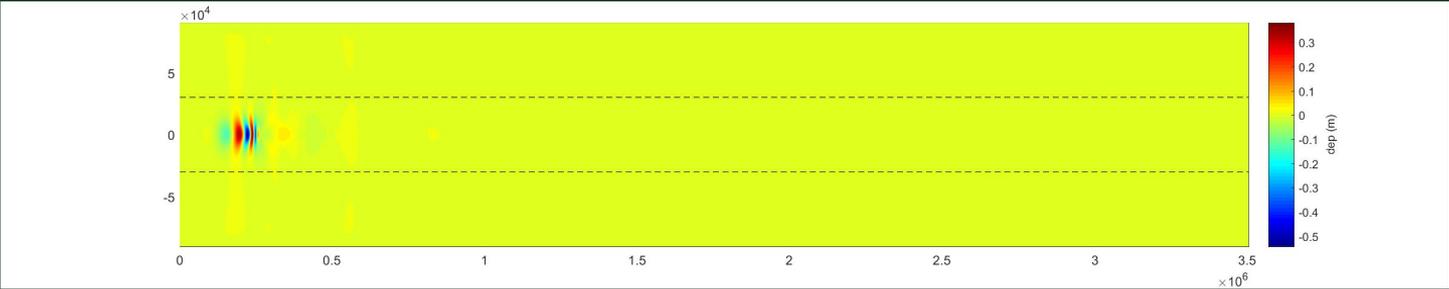
## Total Reflection of Optics

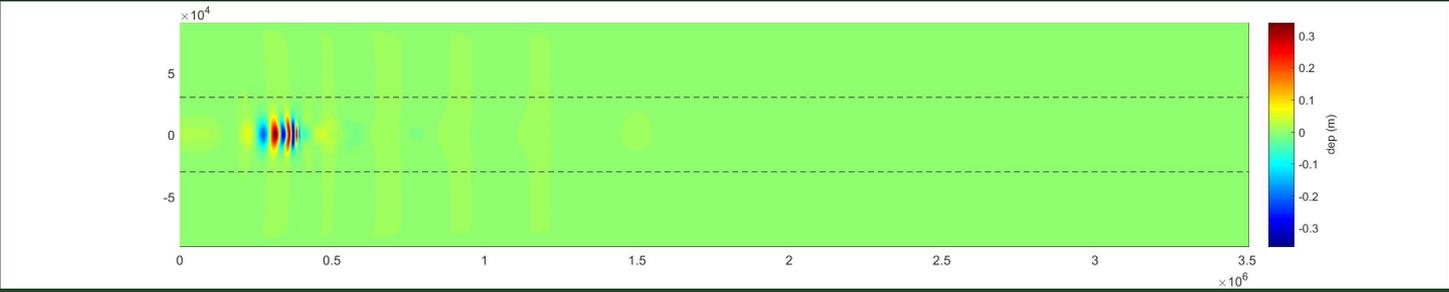
When light passes from denser medium to rarer medium at an angle of incidence greater than the critical angle, all light being reflected back to the denser medium.

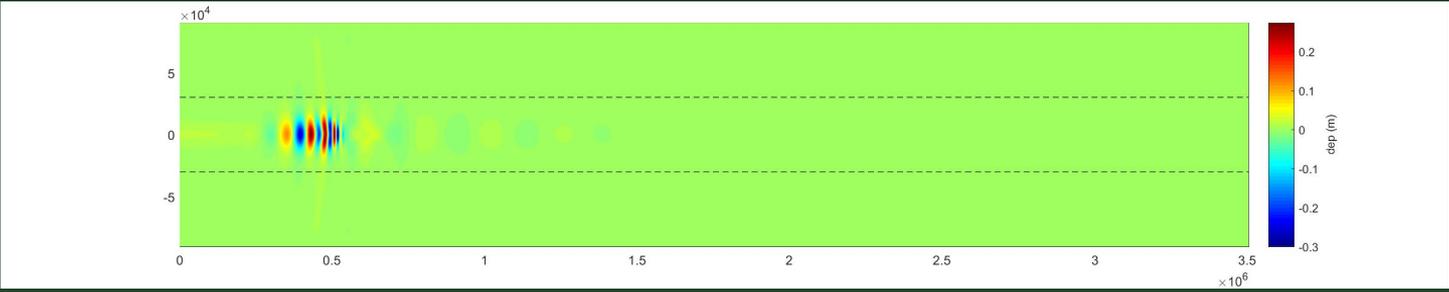


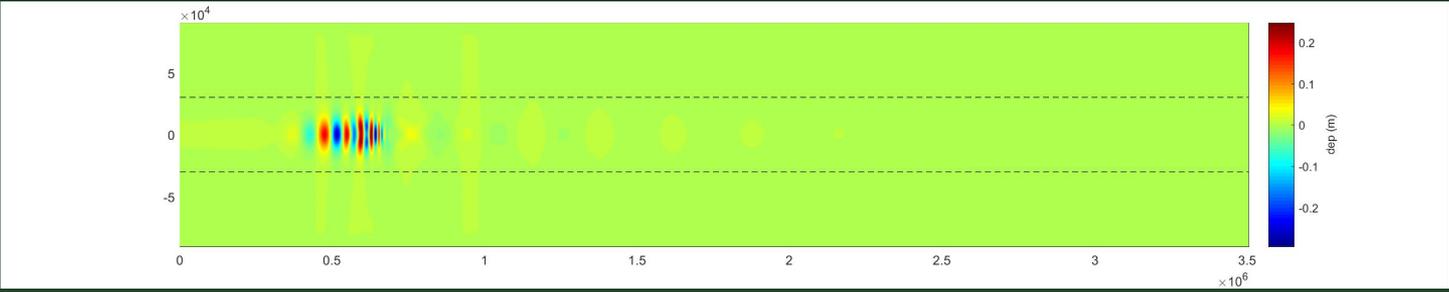


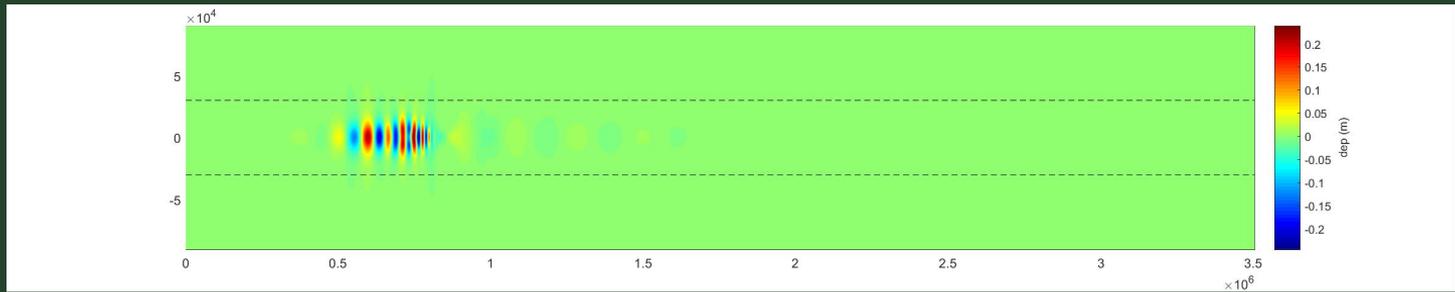


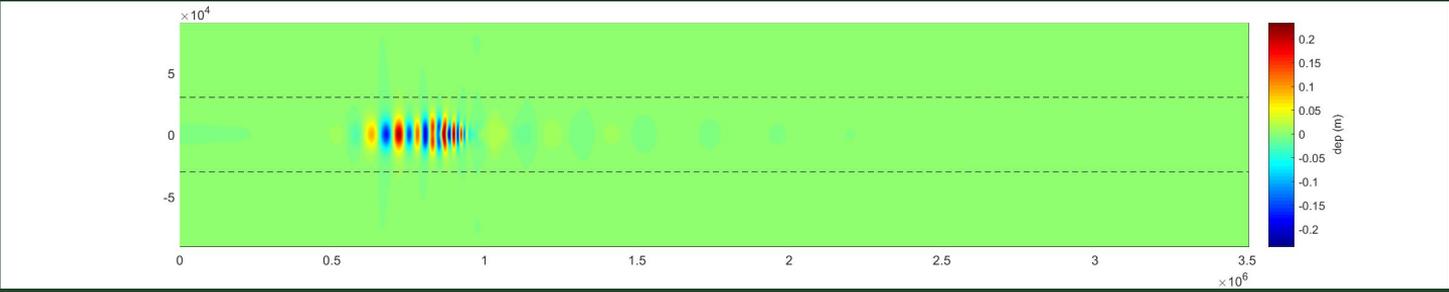


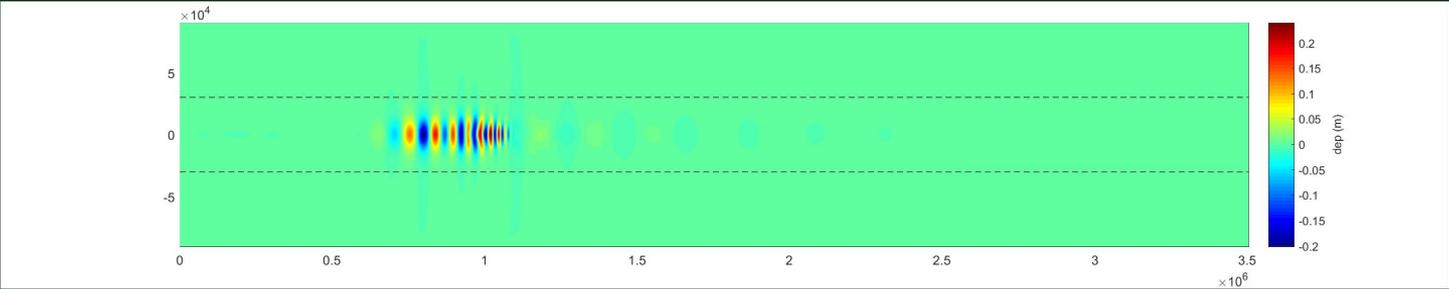


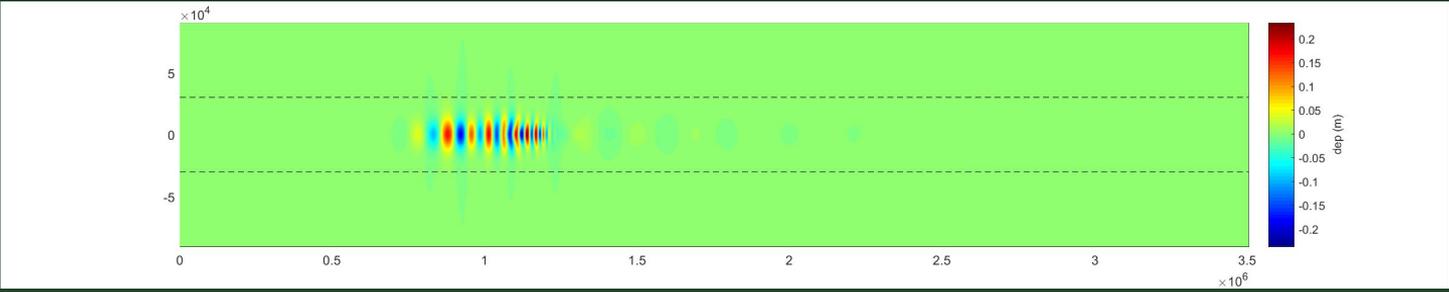


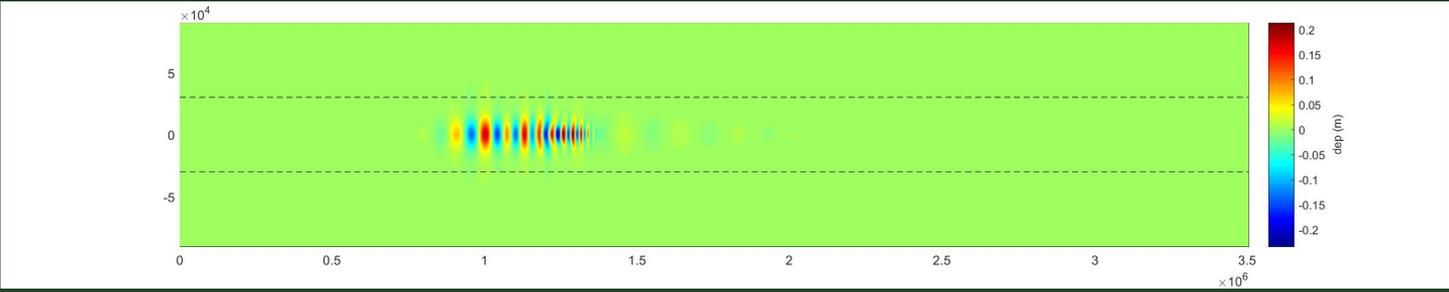


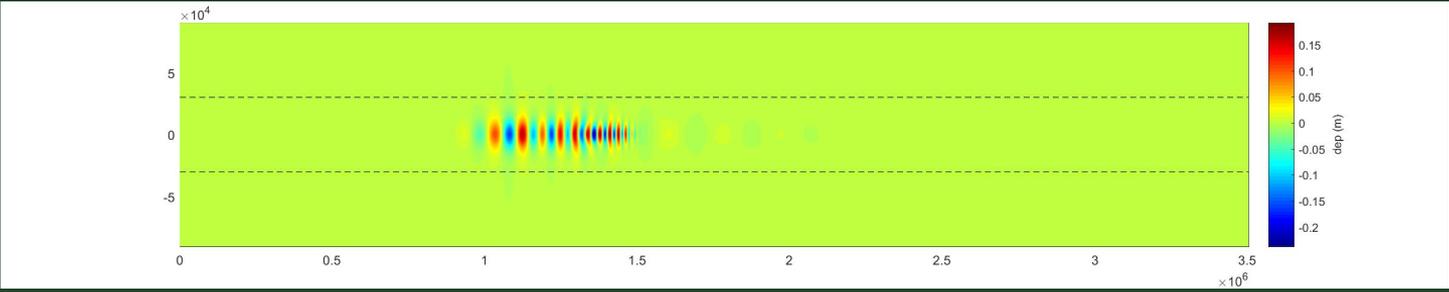


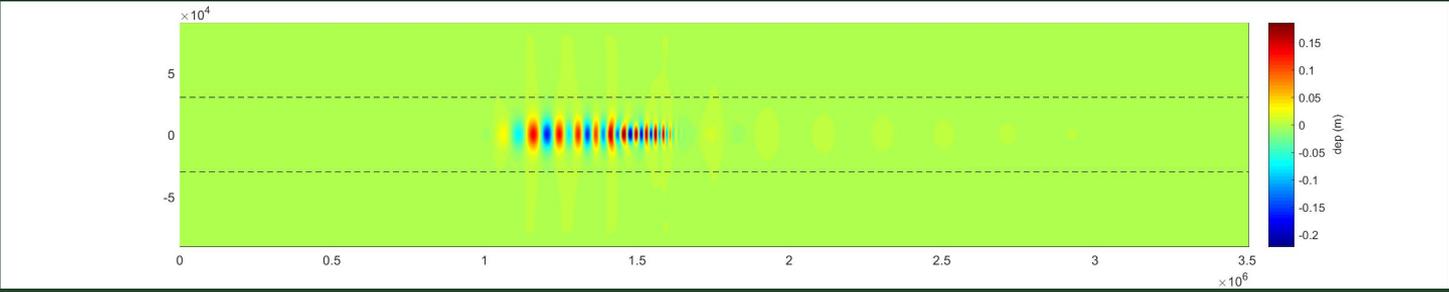


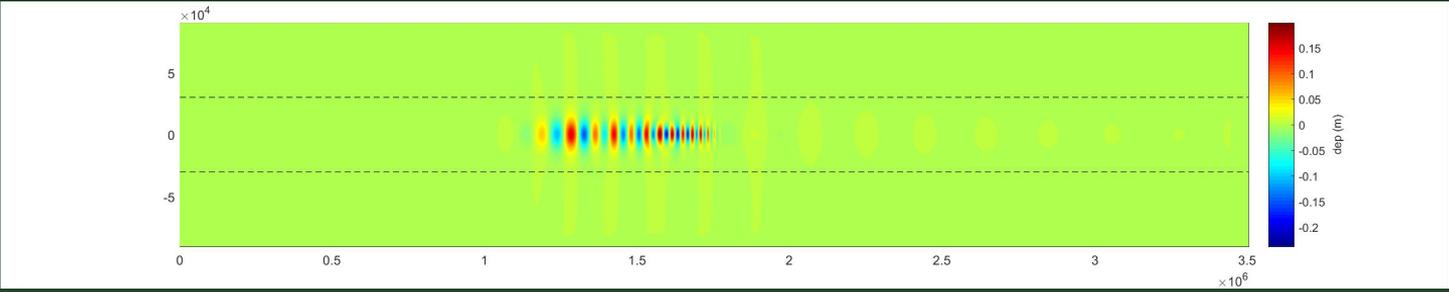


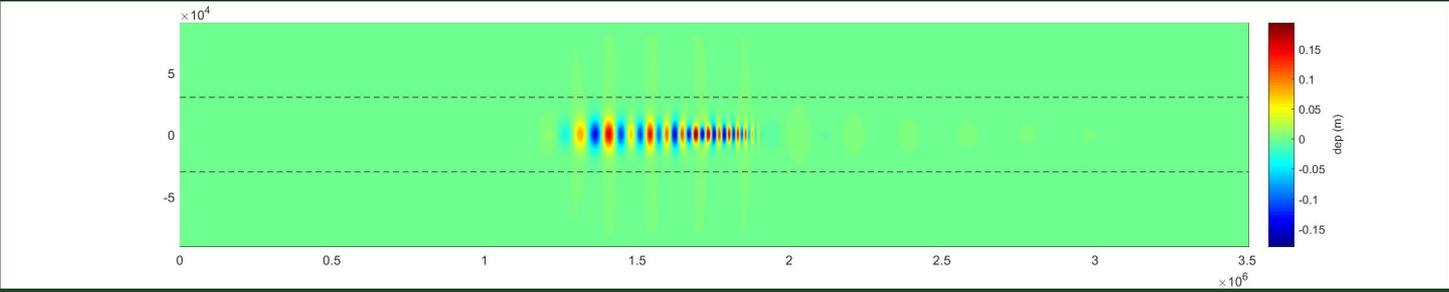


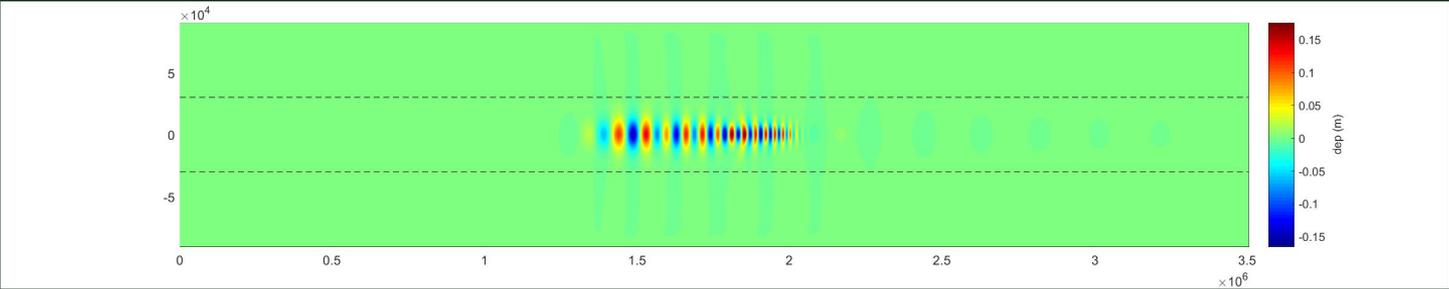


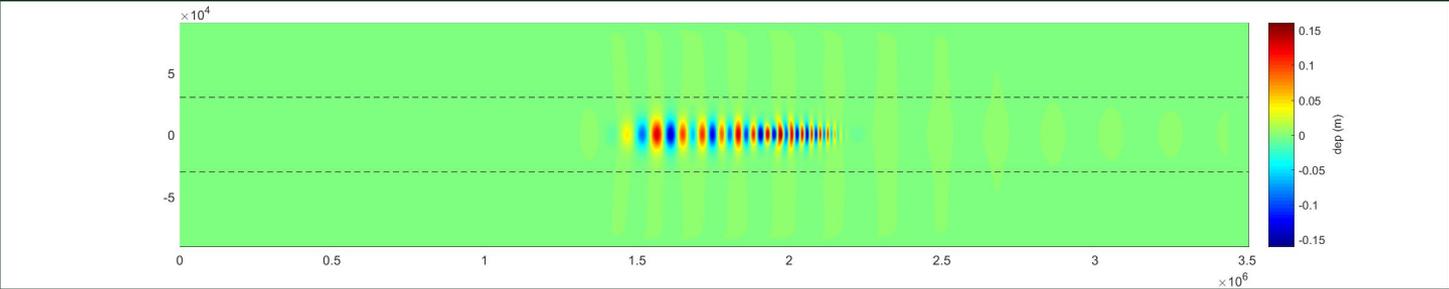


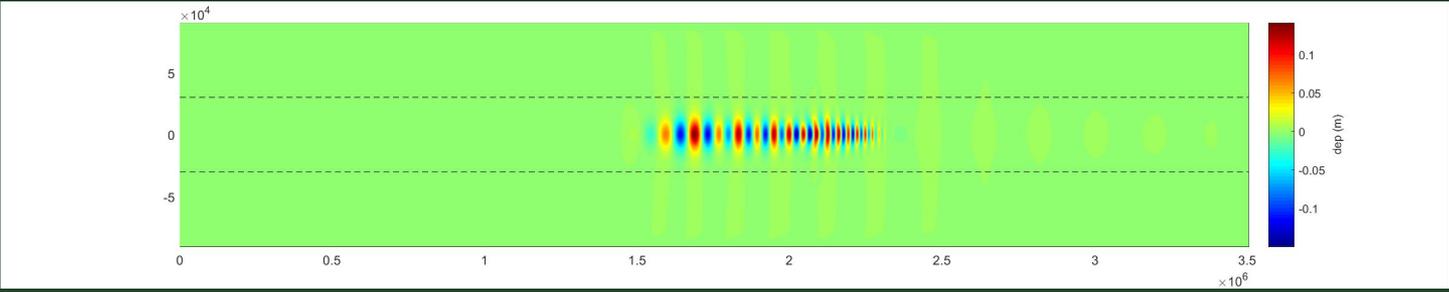


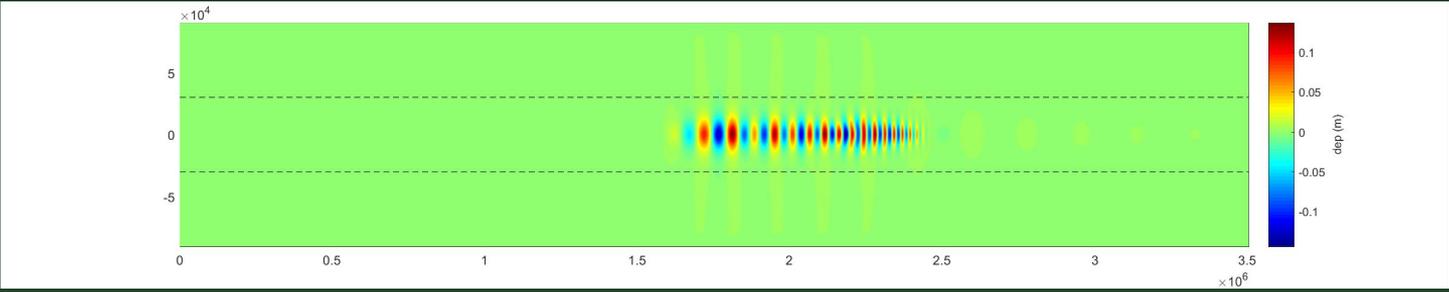


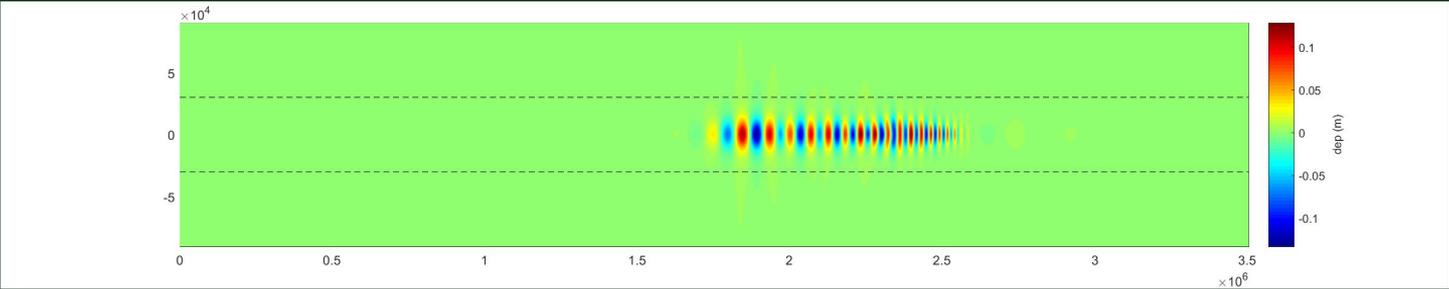


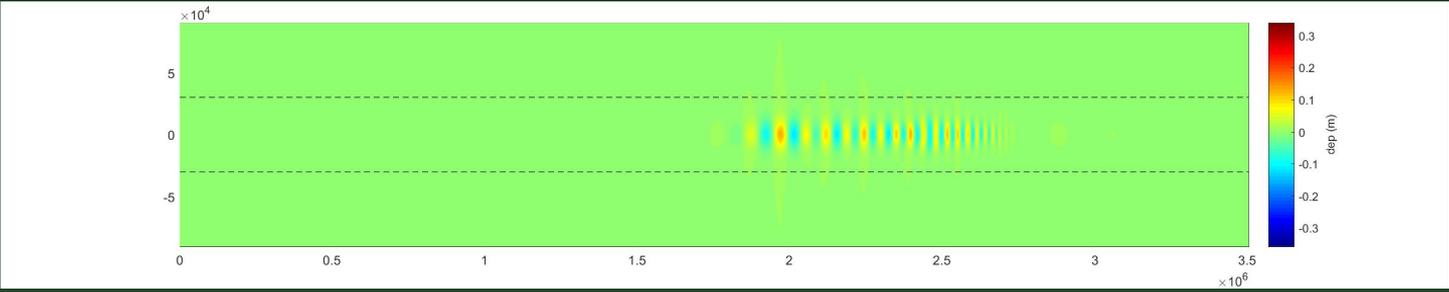


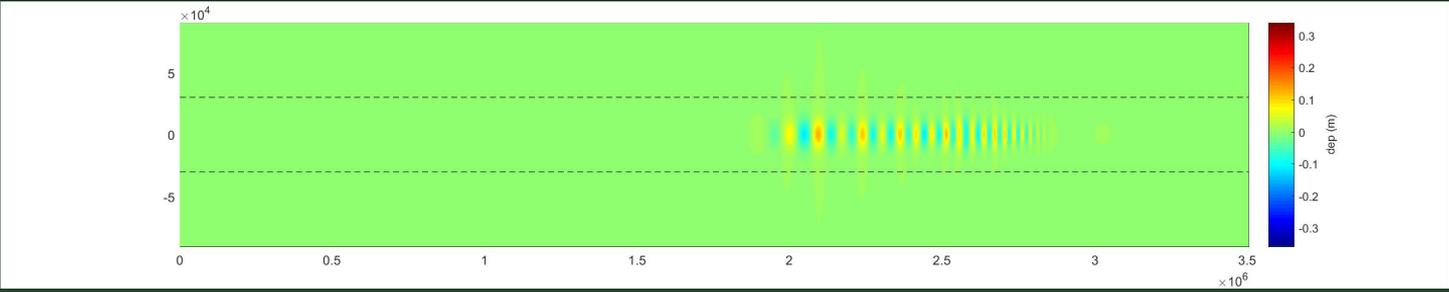


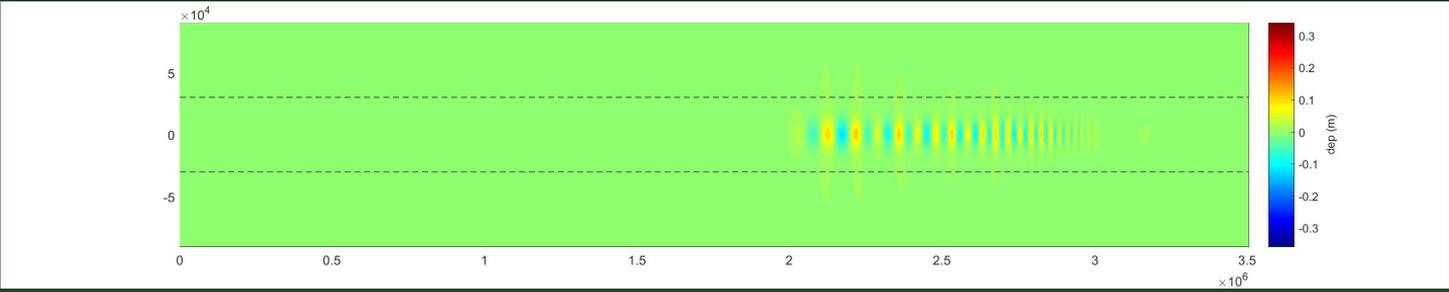


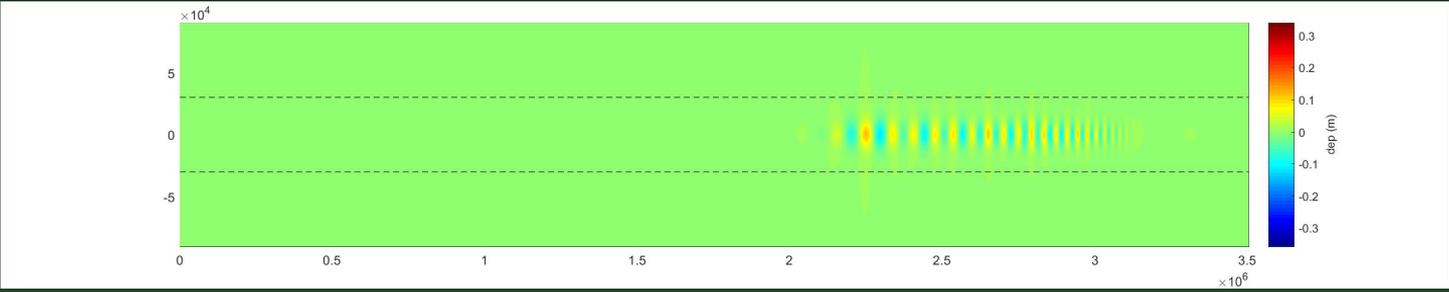


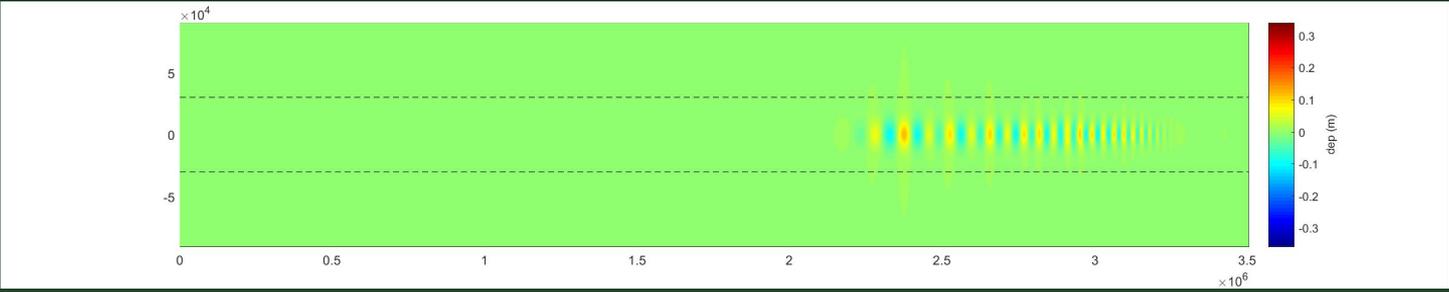


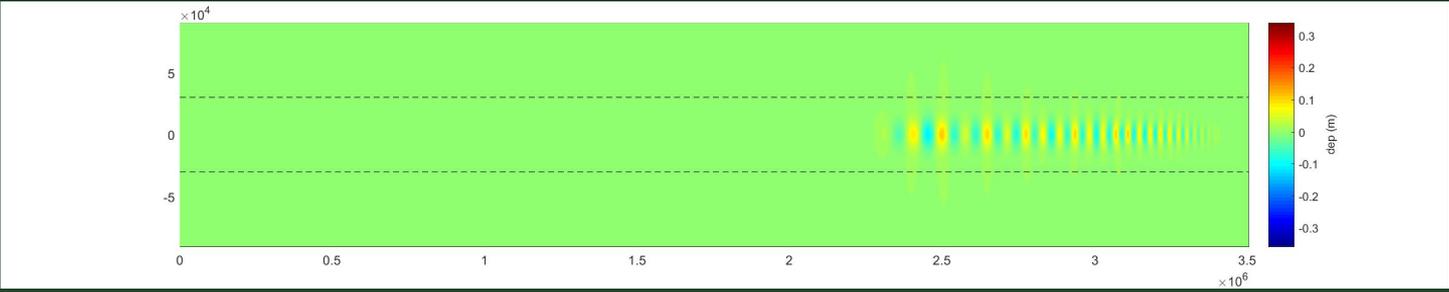


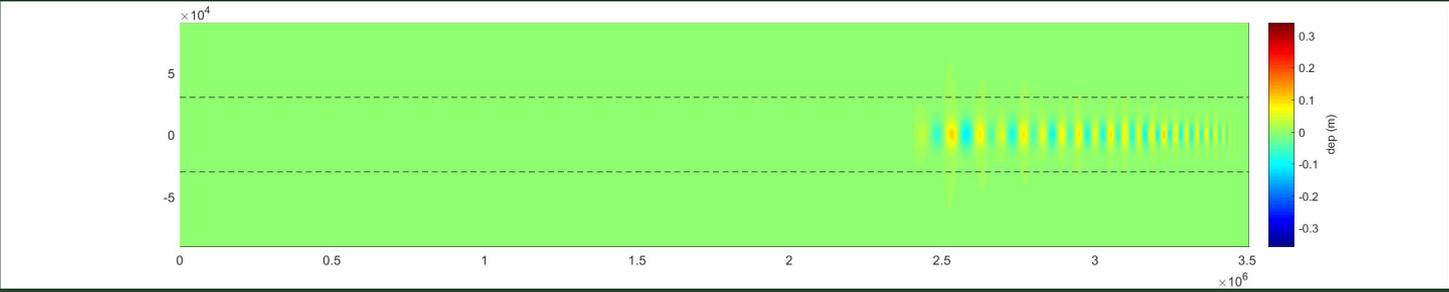


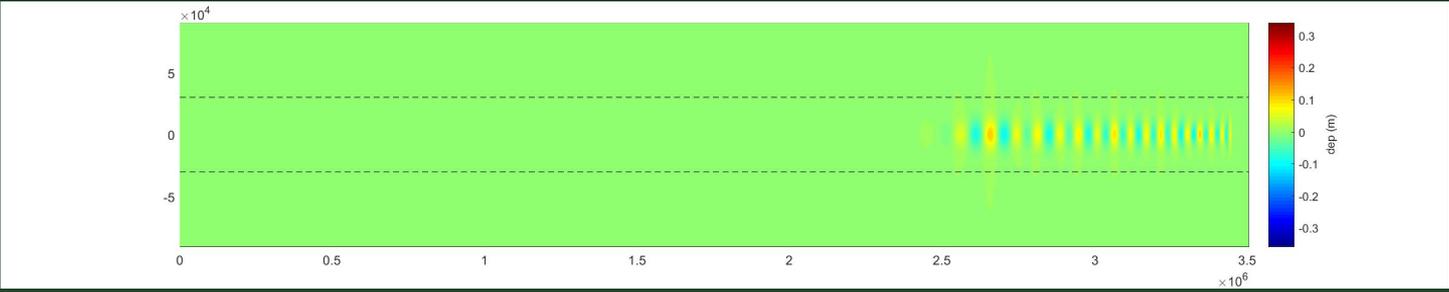


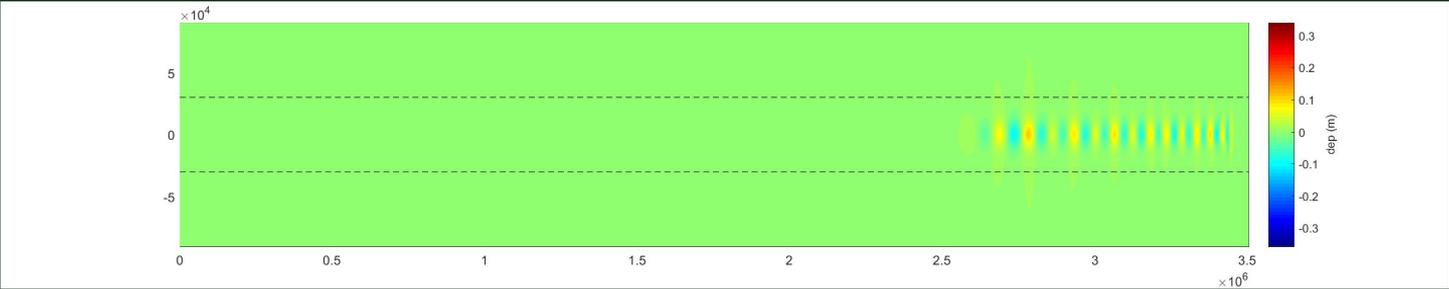


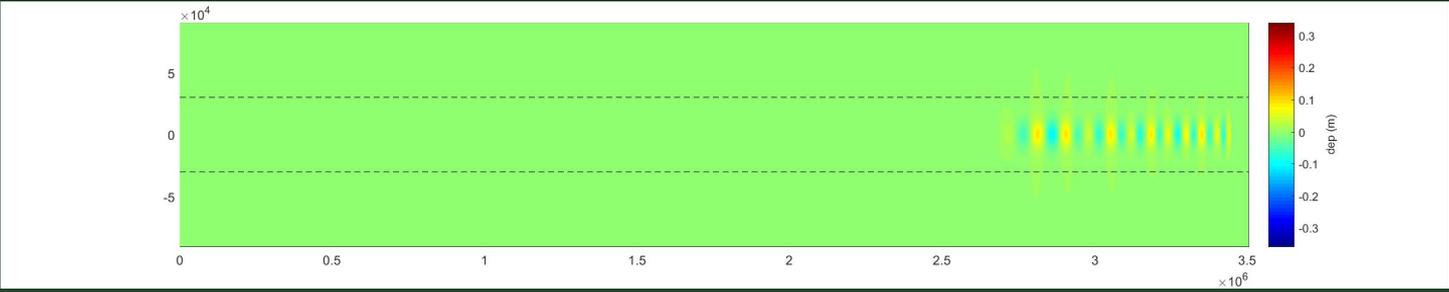


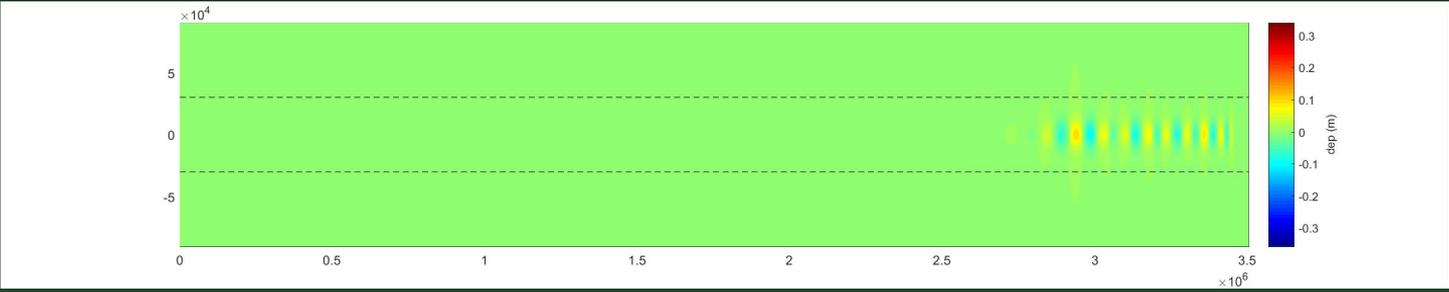


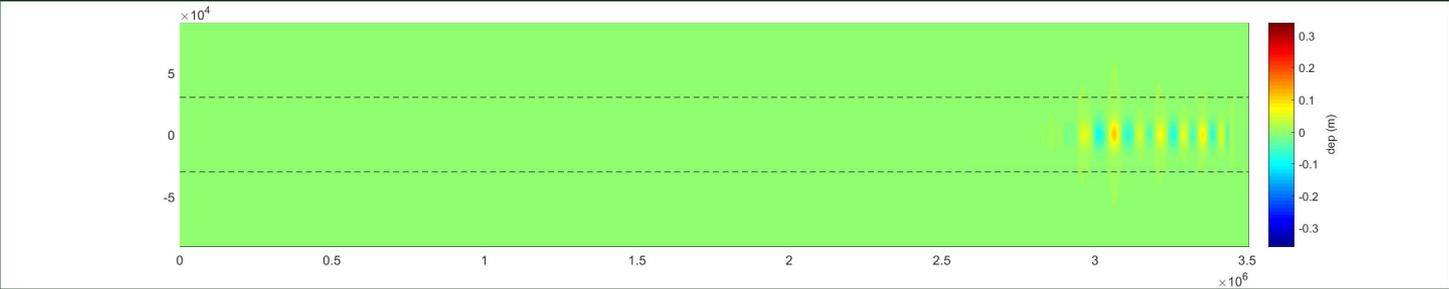


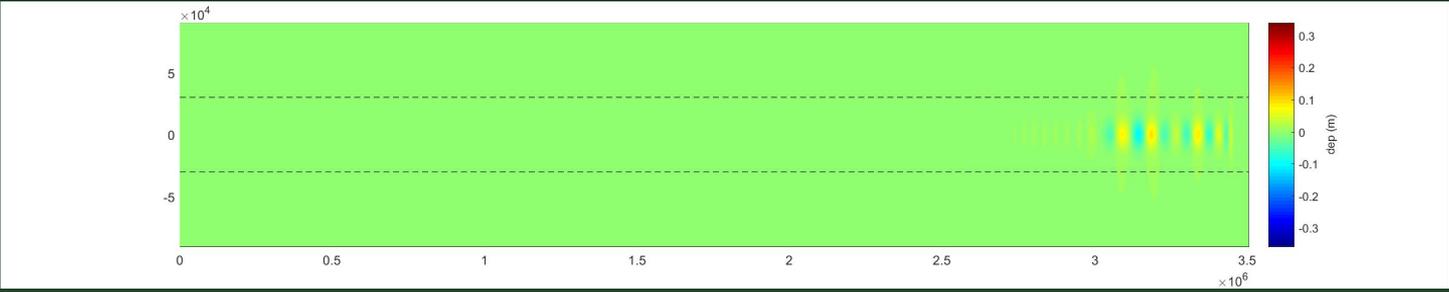




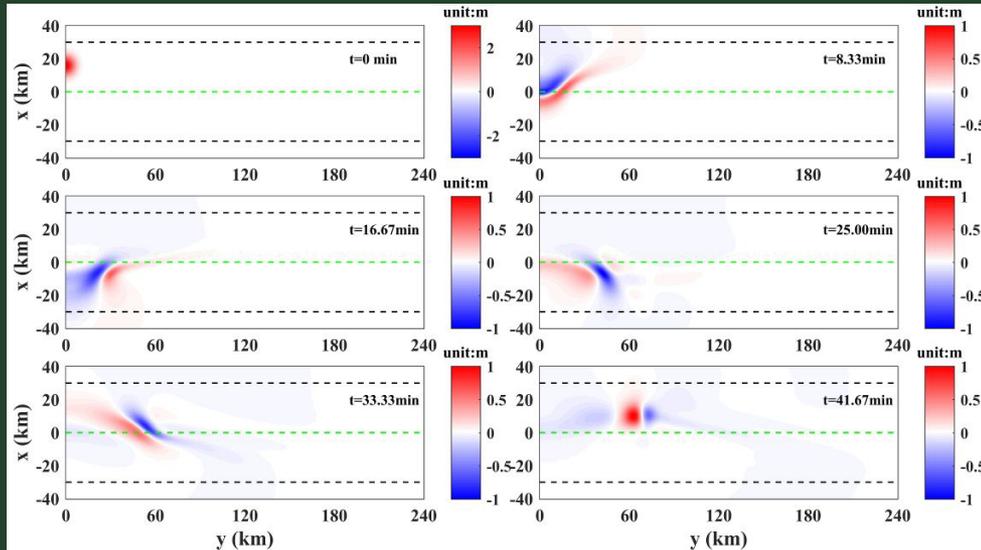
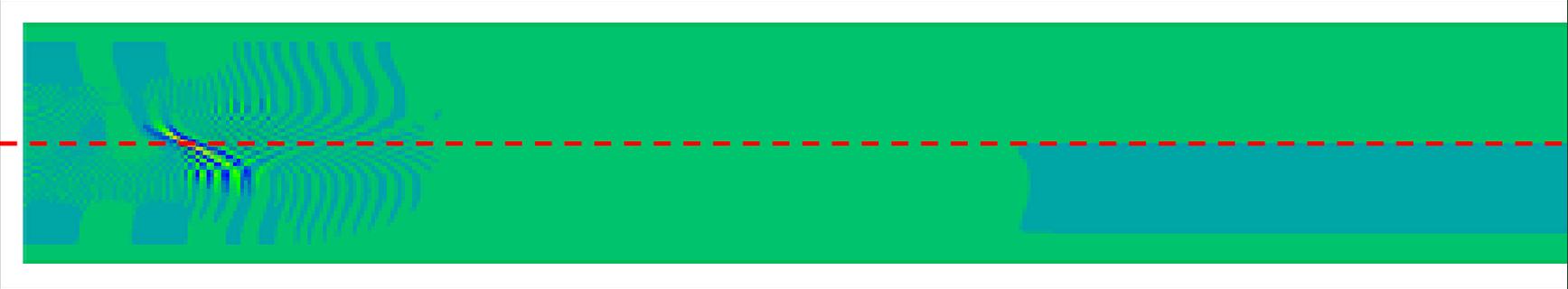






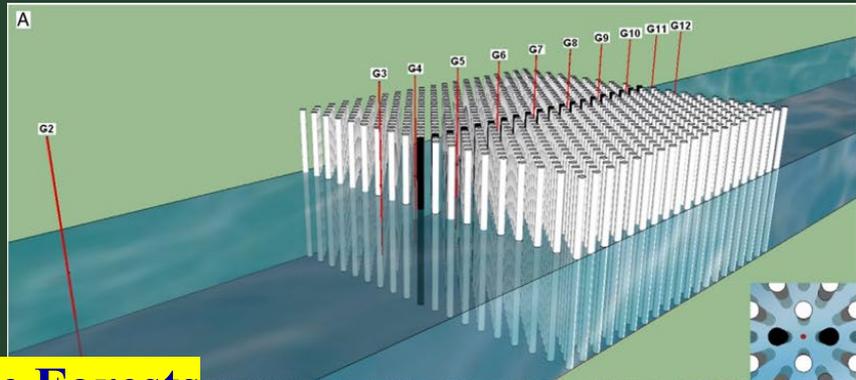


# 4 Trapped Waves over Ridges

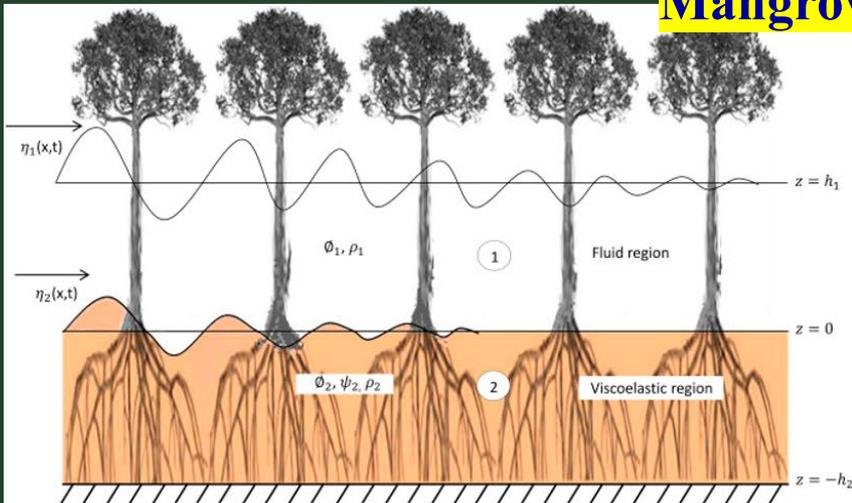




# 5 Further Research



## Mangrove Forests



# 5 Further Research

## Damage induced by Floating Debris

### SWEs model

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} + \frac{\partial \mathbf{g}}{\partial y} = \mathbf{s} \quad \mathbf{s} = \begin{bmatrix} 0 \\ -\tau_{bx}/\rho - g\eta\partial z_b/\partial x + S_{vx} - S_{px} \\ -\tau_{by}/\rho - g\eta\partial z_b/\partial y + S_{vy} - S_{py} \end{bmatrix}$$

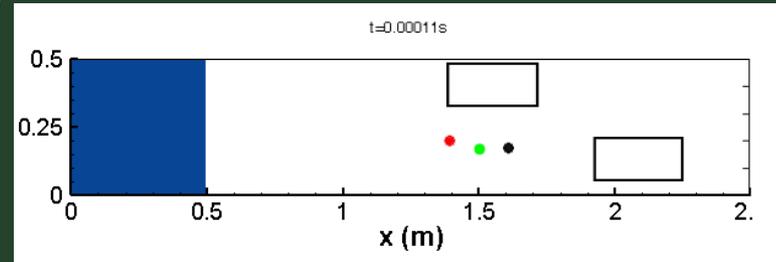
$$\{u, v, h, \eta\} \rightarrow \mathbf{F}_i^f = \mathbf{F}_i^b + \mathbf{F}_i^h$$

### DEM model

$$m_i \frac{d\mathbf{w}_i}{dt} = \mathbf{F}_i^p + \mathbf{F}_i^f + \mathbf{F}_i^g \quad I_i \frac{d\boldsymbol{\omega}_i}{dt} = \mathbf{T}_i^p + \mathbf{T}_i^f$$



$$S_p = p_i^h / \rho$$



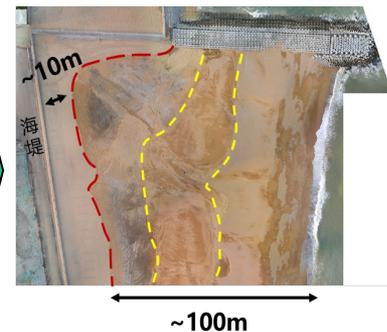
# 5 Further Research

**Typhoon Anbi (2018.07 )**

## Dissipation by Sand Beaches



**Typhoon Lichema (2019.08 )**



Rizhao, Shandong Province, China



*THANK YOU FOR  
YOUR ATTENTION*

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